National Science and Mathematics Olympiad
NSMO

Mathematics 1

Cities and Governorates Competition

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Written By

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Table of Contents

Introduction

Our exceptional sons and daughters,

We are delighted to congratulate you on successfully completing the Cities and Governorates stage and qualifying for the General Administrations stage—an important, advanced step on your path toward mathematical challenge and innovation.

This resource packet is designed to expand your understanding across the four main branches of mathematics: Combinatorics, Geometry, Algebra, and Number Theory. We will focus on advanced concepts in counting, geometric visualization, linear equations, and the principles of distribution and multiplication in Number Theory.

This stage aims to refine your skills in analytical thinking and help you connect mathematical concepts to one another, applying them effectively in various problem situations.

This resource packet is a valuable opportunity to deepen your understanding of mathematical patterns and to use logical reasoning for justification and solving problems using organized methods.

We are confident in your abilities and look forward to seeing you excel in this crucial phase of your journey toward excellence.

The Scientific Team for the National Science and Mathematics Olympiad (NSMO) – Mathematics Track

First Unit: ALGEBRA

1-Integers and Their Properties

1-1 Sets of Numbers:

Set of Natural Numbers:

$$\boldsymbol{N} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \dots\}$$

Set of Whole Numbers:

$$\boldsymbol{W} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \dots\}$$

Set of Integers:

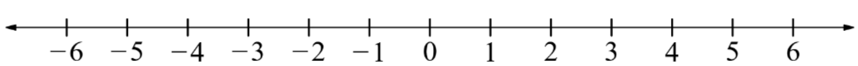
$$\boldsymbol{Z} = \{\dots, -\mathbf{3}, -\mathbf{2}, -\mathbf{1}, \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \dots\}$$

Integers can be divided into three groups:

- Positive integers.
- Zero.
- Negative integers.

Note that the number $+\mathbf{1}$ is usually written simply as $\mathbf{1}$, meaning that the positive sign is often omitted and not pronounced. On the other hand, the negative sign is written and pronounced. Also note that zero is neither positive nor negative.

1-2 The Number Line:



A number line is used to illustrate the order of integers.

As we move to the right, the value of the integer increases, and as we move to the left, the value decreases.

This means that any positive integer is greater than any negative integer, and zero is greater than any negative number but less than any positive number.

For the numbers $\mathbf{5}$ and $-\mathbf{5}$, each is the additive inverse (or opposite) of the other.

In general, for any integer a, its opposite −a is its additive inverse, while zero is the additive inverse of itself.

On the number line, an integer and its additive inverse are represented by two points that are the same distance from zero but on opposite sides of it.

1-3 The Absolute Value of an Integer:

The absolute value of an integer $\boldsymbol{a}$ is denoted by $|\,\boldsymbol{a}\,|$.

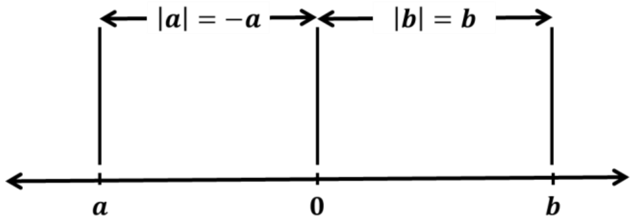
For example:

$$|\,5\,| = 5 \quad , \quad |\,-5\,| = 5 \quad , \quad |\,0\,| = 0$$

Geometrically, each integer is represented by a point on the number line.

The expression $|\,\boldsymbol{a}\,|$ represents the distance between the point representing the integer $\boldsymbol{a}$ and the (zero) on the number line.

In general, $|\,\boldsymbol{a} - \boldsymbol{b}\,|$ represents the distance between the points representing the two integers $\boldsymbol{a}$ and $\boldsymbol{b}$.



When finding the absolute value of an algebraic expression, if the result is negative, it is made non-negative by removing the negative sign (−).

1-4 Fundamental Rules of Addition, Subtraction, Multiplication, and Division:

Commutative Property:

- $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a}$
- $\boldsymbol{ab} = \boldsymbol{ba}$

Associative Property:

- $(\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c} = \boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c})$
- $(\boldsymbol{ab})\boldsymbol{c} = \boldsymbol{a}(\boldsymbol{bc})$

Distributive Property:

- $\boldsymbol{a(b+c) = (b+c)a = ab+ac}$
- $\boldsymbol{a(b-c) = (b-c)a = ab-ac}$

Closure Property:

The closure property holds for addition, subtraction, and multiplication, but not for division.

For any two integers $\boldsymbol{a}$ and $\boldsymbol{b}$, the results of $\boldsymbol{a}+b$, $\boldsymbol{a}-b$, and $\boldsymbol{a}\times b$ are always integers.

However, $\boldsymbol{a}\div\boldsymbol{b}$ is not necessarily an integer.

Additive Identity Property:

For any integer $\boldsymbol{a}\in\mathbb{Z}$, the number $\boldsymbol{0}$ is the additive identity, because:

- $\boldsymbol{a+0=0+a=a}$

Additive Inverse Property:

For every integer $\boldsymbol{a}\in\mathbb{Z}$, there exists an integer $\boldsymbol{-a}\in\mathbb{Z}$ such that:

- $\boldsymbol{a+(-a)=(-a)+a=0}$

1-5 Exponents:

An exponent is used to represent repeated multiplication.

For example:

$$\boldsymbol{2\times 2\times 2 = 2^3}$$

In general:

$$\boldsymbol{a^n = \overbrace{a\times a\times a\times\ldots\times a}^{n\ times}}$$

Properties of Exponents (Powers):

If $x, y > 0$ and $m, n \in \mathbb{Z}$, then:

$(1)\ x^m \cdot x^n = x^{m+n}$ $(2)\ \frac{x^m}{x^n} = x^{m-n}$ $(3)\ (x^m)^n = x^{mn}$

$(4)\ (xy)^n = x^n y^n$ $(5)\ \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ $(6) x^0 = 1$

$(7)\ x^{-1} = \frac{1}{x}$

1-6 Rule for Removing Parentheses:

For any two integers x and y:

$(1)\ x + (y) = x + y \quad , \quad x + (-y) = x - y$

$(2)\ x - (y) = x - y \quad , \quad x - (-y) = x + y$

$(3)\ x(-y) = (-x)y = -xy \quad , \quad (-x)(-y) = xy$

$$(4) \begin{cases} (-1)^n = -1 & if\ n\ is\ an\ \mathbf{odd}\ integer. \\ (-1)^n = 1 & if\ n\ is\ an\ \mathbf{even}\ integer. \end{cases}$$

Examples:

(1) Find the value of each expression:

$a)\ (-2) + 12 =$ $b)\ (-3) + (-6) + 5 =$ $c) - 3 - 11 - 31 =$

$d)\ (-5) \times (-4) =$ $e)\ (-4) \times 8 =$ $f)\ (-1) \times 2 \times 2 =$

Solution:

$a)\ (-2) + 12 = 10$

$b)\ (-3) + (-6) + 5 = [(-3) + (-6)] + 5$
$= (-9) + 5 = -4$

$\boldsymbol{c})\ -3-11-31=-(3+11+31)=-45$

$\boldsymbol{d})\ (-5)\times(-4)=20$

$\boldsymbol{e})\ (-4)\times 8=-32$

$\boldsymbol{f})\ (-1)\times 2\times 2=-4$

(2) Find the value of each expression:

$\boldsymbol{a})\ |-2|+(-2)=$

$\boldsymbol{b})\ (-3)^2-3=$

$\boldsymbol{c})\ (a^2)^3\times a^3=$

$\boldsymbol{d})\ \dfrac{x^3\times x}{x^2}=$

Solution:

$\boldsymbol{a})\ |-2|+(-2)=2-2=0$
$\boldsymbol{b})\ (-3)^2-3=9-3=6$
$\boldsymbol{c})\ (a^2)^3\times a^3=a^6\times a^3=a^9$
$\boldsymbol{d})\ \dfrac{x^3\times x}{x^2}=\dfrac{x^4}{x^2}=x^2$

(3) Simplify:

$$3a+\{-4b-[4a-7b-(-4a-b)]+5a\}$$

Solution:

$$3a+\{-4b-[4a-7b-(-4a-b)]+5a\}=3a+\{-4b-[4a-7b+4a+b]+5a\}$$

$$=3a+\{-4b-[8a-6b]+5a\}$$

$$=3a+\{-4b-8a+6b+5a\}$$

$$=3a+\{2b-3a\}$$

$$=2b+(3a-3a)$$
$$=2b$$

Exercises:

(1) Find the value of each expression:

$\boldsymbol{a})\ (-4) + 9 =$
$\boldsymbol{b})\ -42 \div 7 =$
$\boldsymbol{c})\ (2)^5 =$
$\boldsymbol{d})\ (-4)^3 =$
$\boldsymbol{e})\ (-5)^2 =$
$\boldsymbol{f})\ |0| =$
$\boldsymbol{g})\ (-4) \times (-8) =$
$\boldsymbol{h})\ -8 - (-5) =$
$\boldsymbol{i})\ -1 - 4 + 7 =$
$\boldsymbol{j})\ 2 \times 4 + 6 \times 5 =$
$\boldsymbol{k})\ |-6| =$
$\boldsymbol{m})\ |-3| - |-7| =$

(2) Find the value of each expression:

$\boldsymbol{a})\ a^2 \times a^5 =$
$\boldsymbol{b})\ x^7 \div x^3 =$
$\boldsymbol{c})\ (a^3)^4 =$
$\boldsymbol{d})\ (x^2)^3 \times (x^4)^5 =$
$\boldsymbol{e})\ \dfrac{a^3 \times a^7}{a^2 \times a^6} =$
$\boldsymbol{f})\ \dfrac{a^4 \times a^5}{a^3 \times a^6} =$

(3) Compute:

$\boldsymbol{a})\ (-7) + (-12) - (-14) - (-15) - (-18) - (-38)$

$\boldsymbol{b})\ (-5)^2 + |-6| - (-1)^{1447}$

Challenge Problems:

(1) Compute:

$$-1-(-1)^1-(-1)^2-(-1)^3-\cdots-(-1)^{99}-(-1)^{100}$$

(2) Find the value of:

$$1234 \times 9999 = \cdots$$

(3) Simplify:

$$5\{(2a-3)-[7(4a-1)-20]\}-(3-8a)$$

(4) A satellite completes one full orbit around the Earth every 7 hours.

How many times will the satellite orbit the Earth in one week?

(5) Six fair dice were rolled. The sum of the numbers shown on all six faces is 32.

What is the smallest possible number that could appear on one of the dice?

(6) Compute:

$$1-2+3-4+\ldots-100+101$$

2-Rational Numbers $\mathbb{Q}$

A rational number is any number that can be written in the form

$$\frac{a}{b}$$

where a and b are integers, and $b \neq 0$.

The number a is called the numerator, and b is called the denominator.

From this definition, we can conclude that every integer is a rational number.

For example:

$$0 = \frac{0}{1}, -4 = \frac{-4}{1}, 5 = \frac{5}{1}$$

Also, the mixed number

$$5\frac{1}{2} = \frac{11}{2}$$

is a rational number because it can be written as a fraction.

And so on…

Terminating decimals are rational numbers. For example:

$$0 \cdot 3 = \frac{3}{10}, 1.7 = \frac{17}{10}, 2.03 = \frac{203}{100}$$

Non-terminating decimals are of two types:

Repeating decimals:

$$0.333 \ldots = 0.\overline{3}, \quad 0.521521521 \ldots = 0.\overline{521}$$

These are rational numbers because their digits repeat in a predictable pattern.

Non-repeating decimals:

$$0.345674215677589\ldots$$

These are irrational numbers, because their digits continue infinitely without any repeating pattern.

Note:

The approximate ratio known as pi (π) is an irrational number.

It is approximately equal to 3.14, or sometimes represented as

$$\frac{22}{7},$$

which is only an approximation.

2-1 Properties of Addition and Subtraction of Rational Numbers

The properties of addition for rational numbers are the same as those for integers.

But wait—when adding or subtracting two rational numbers, why do we always hear that we must "find a common denominator"? Let's explore this question.

Actually, the answer is not as simple as it seems. Let's take a closer look.

We all know that the sum of three camels and five camels is eight camels. In general:

$$3x + 5x = 8x$$

where the symbol x represents the same quantity.

Now, suppose we divide a pizza into seven equal and identical parts. Each part is called one-seventh and is written as:

$$\frac{1}{7}$$

However, when we use symbols, things may appear different:

$$2\left(\frac{1}{7}\right) + 3\left(\frac{1}{7}\right) = 5\left(\frac{1}{7}\right)$$

Since $2\left(\frac{1}{7}\right)$ can be written as $\frac{2}{7}$, we can express the equation in a simpler form:

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

Here, we added numerators because the denominators are the same.

(Notice that many students mistakenly try to add denominators as well — but denominators must remain the same!)

By similar reasoning, you can conclude that subtracting two rational numbers also requires the same denominator.

But what if the denominators are different?

How do we add or subtract rational numbers then?

We'll soon find the answer after studying the next property of rational numbers.

2-2 The Property of Equivalent Rational Numbers

If $\frac{a}{b}$ is a rational number and k is a nonzero integer, then:

$$\frac{a}{b} = \frac{a \cdot k}{b \cdot k} = \frac{a \div k}{b \div k}$$

This means that a rational number can be written in many equivalent forms.

For example:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10} = \cdots$$

$$\frac{18}{24} = \frac{9}{12} = \frac{3}{4}$$

A rational number whose numerator and denominator have no common factor (other than 1) is said to be in simplest form.

The simplest form of a rational number is the most convenient to use in calculations. Therefore, it is always recommended to express any rational number in its simplest form.

Notice: Now You Can Add Two Rational Numbers with Different Denominators!

For example, when adding

$$\frac{2}{3}+\frac{1}{5}$$

we first look for a common multiple of the denominators 3 and 5 (preferably the least common multiple).

It is easy to see that 15 is a multiple of both 3 and 5.

Now, we rewrite the fractions as equivalent fractions with a common denominator:

$$\frac{2\times 5}{3\times 5}+\frac{1\times 3}{5\times 3}$$

That is,

$$\frac{2}{3}+\frac{1}{5}=\frac{10}{15}+\frac{3}{15}$$

and this equals

$$\frac{13}{15}$$

Subtraction is handled in the same way as addition.

2-3 Definition of Multiplication and Division in Rational Numbers

How do we multiply two rational numbers, and what does that mean?

For example, when we ask, *"What is half of 100?"* you immediately answer 50 — as if you divided 100 by 2.

Now, if we ask, *"What is three halves of 100?"* it's not difficult to find the answer: you first divide by 2, then multiply the result by 3.

Let's ask a more complex question:

What is three halves of the number $\frac{1}{7}$?

We need to divide the number by 2, and then multiply the result by 3.

But what is the result of dividing $\frac{1}{7}$ by 2?

$$\frac{1}{7} = \frac{2}{14} = 2\left(\frac{1}{14}\right)$$

So, if the result of dividing $\frac{1}{7}$ by 2 is $\frac{1}{14}$ (that is, only the denominator is multiplied by 2), then multiplying by 3 means making three copies of that result:

$$3\left(\frac{1}{14}\right) = \frac{3}{14}$$

Let's express what we found symbolically:

$$100 \times \frac{3}{2} = 50 \times 3 = 150$$

Similarly,

$$\frac{1}{7} \times \frac{3}{2} = \frac{1 \times 3}{7 \times 2} = \frac{3}{14}$$

In general, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Finally, since division is the inverse operation of multiplication, we can justify the following definition:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

2-4 Properties of Multiplication of Rational Numbers

1. Closure Property:

   The product of any two rational numbers is also a rational number.

2. Commutative Property:

Changing the order of the factors does not change the product.

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$$

3. Associative Property:

The way the factors are grouped does not affect the product.

4. Multiplicative Identity:

The number 1 is the multiplicative identity because multiplying any rational number by 1 does not change its value.

5. Multiplicative Inverse:

For every non-zero rational number $\frac{a}{b}$, there exists a number $\frac{b}{a}$ called its multiplicative inverse, such that:

$$\frac{a}{b} \times \frac{b}{a} = \frac{b}{a} \times \frac{a}{b} = 1$$

6. Distributive Property of Multiplication over Addition and Subtraction

2-5 Writing a Repeating Decimal as a Rational Number

Example:

Write the repeating decimal $\mathbf{0.\overline{7}}$ as a rational number.

Solution:

Let

$$x = 0.\bar{7} = 0.7777 \ldots$$

Multiply both sides by 10:

$$10x = 7.7777 \ldots = 7 + 0.7777 \ldots$$

Now we can write:

$$10x = 7 + x$$

Subtract x from both sides:

$$9x = 7$$

Finally, divide both sides by 9 to find:

$$x = \frac{7}{9}$$

Practice:

Write the repeating decimal $0.\overline{17}$ as a rational number.

Hint: Follow the same steps as in the example, but this time you will need to multiply by 100 instead of 10.

A Quick Method for Writing a Repeating Decimal as a Rational Number

(The proof is not difficult.)

We can write a repeating decimal as a fraction directly, provided that the repeating part starts immediately after the decimal point.

To do this, place the repeating digits in the numerator, and write as many nines in the denominator as there are repeating digits.

For example:

$$\mathbf{0.\overline{7} = \frac{7}{9}}, \qquad \mathbf{0.\overline{73} = \frac{73}{99}}, \qquad \mathbf{0.\overline{732} = \frac{732}{999}}, \qquad \text{and so on.}$$

This method can also be extended to write any other repeating decimal as a rational number. For example:

$$\begin{aligned}\mathbf{0.1\overline{7}} &= \mathbf{0.1 + 0.0\overline{7}} \\ &= \mathbf{\frac{1}{10} + \frac{1}{10} \times \frac{7}{9}} \\ &= \mathbf{\frac{1}{10} + \frac{7}{90}} \\ &= \mathbf{\frac{9}{90} + \frac{7}{90}} \\ &= \mathbf{\frac{16}{90}} \\ &= \mathbf{\frac{8}{45}}\end{aligned}$$

Exercises:

(1) Find the value of each of the following:

$(\boldsymbol{a})\ \frac{2}{5}+\frac{1}{6}=$

$(\boldsymbol{b})\ \frac{1}{2}-\frac{1}{3}=$

$(\boldsymbol{c})\ \frac{-3}{8}\times\frac{4}{9}=$

$(\boldsymbol{d})\ \frac{-3}{5}-\left(-\frac{1}{2}\right)$

$(\boldsymbol{e})\ 1\div 3\div 4\div 5=$

$(\boldsymbol{f})\ \frac{1}{9}-\frac{1}{10}=$

$(\boldsymbol{g})\ (-3)\div 4\times 6\div(-5)=$

$(\boldsymbol{h})\ \frac{-6}{35}\div\frac{2}{7}=$

$(\boldsymbol{i})\ \left(\frac{-1}{2}\right)^5=$

(2) Write each of the following repeating decimals as a rational number:

$\boldsymbol{a})\ 0.\overline{2}$ $\boldsymbol{b})\ 0.\overline{37}$ $\boldsymbol{c})\ 1.8\overline{23}$

(3) Majed claims that dividing by an integer is equivalent to multiplying by its multiplicative inverse.

Can you prove or disprove this claim?

(4) Saleh claims that between any two rational numbers, there is always another rational number.

He also states that this property is called the density of rational numbers.

Can you give an example that supports his claim, or provide a counterexample if you think otherwise?

(5) Arrange the following numbers in ascending order:

$$-0.3\ ,-0.23\ ,-\frac{1}{3}$$

Challenge Problems:

(1) Find a fraction that is greater than $\frac{1}{6}$ and less than $\frac{1}{3}$, with a denominator of 15.
(2) Saad divided a number N by 8 and got 0.25, while Khaled multiplied the same number N by 8. What result will Khaled obtain?
(3) Haitham went jogging outside his house. He has already completed $\frac{3}{5}$ of the second half of his route. What fraction of the entire route has he completed?
(4) Compute: $(1-\frac{1}{2})+(\frac{1}{2}-\frac{1}{3})+(\frac{1}{3}-\frac{1}{4})+\ldots+(\frac{1}{9}-\frac{1}{10})$
(5) Compute: $\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdots\cdots\frac{79}{80}$
(6) Adel has a card containing 14 digits. If the sum of any 3 consecutive digits (starting from the left) is 20, he wrote down the digits on his card as shown below. What number should be written in the square labeled A? | A | | 7 | | | | | | | | | 7 | | 4 |
(7) Today is the birthday of Laila and her two daughters, Shahd and Amal. Laila is 32 years old, Shahd is 4 years old, and Amal is 1 year old. How old will Laila be when her age becomes equal to the sum of Shahd's and Amal's ages?

(8) My uncle lives far away in an isolated place, and his letters to us always contain riddles.

In one of his letters, he told us that there are three local teams in his area: the Ants (A), the Bees (B), and the Cats (C).

Each year, they play a local league, where every team plays against each of the others at most once.

The league table for this year began as follows:

Team	Played	Won	Drawn	Lost	Goals For	Goals Against
A	1	0	0	1	4	2
B	2	1	1	0	2	2
C	2	1	0	1	3	1

When we pointed out that these numbers were impossible, my uncle admitted that every number in the table was incorrect,

but that each number should either have 1 added to it or 1 subtracted from it.

Find the correct table, and explain clearly how you deduced your results.

Second Unit: Geometry

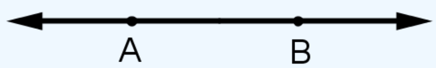
1-Points, Lines, and Angles

Geometry starts with three simple objects: points, lines, and angles. They are so simple that you already see them every day, but so powerful that they can build entire worlds. Let us start slowly and make friends with them.

Definitions:

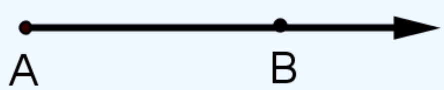
Definition 1.1

A *line* is a perfectly straight path that extends without end in both directions. It has no thickness and no boundaries.



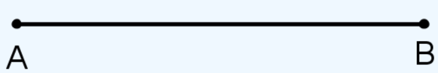
Definition 1.2

A *ray* begins at one point (called its *endpoint*) and extends forever in one direction. It is like a flashlight beam: it starts at the bulb and travels infinitely forward.



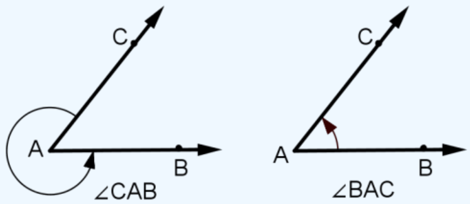
Definition 1.3

A *segment* is the part of a line between two points. It has a beginning and an end, just like a stick you can hold.



Definition 1.4

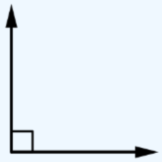
An *angle* is formed when two rays meet at a common endpoint (the *vertex*). Angles can be measured in two directions: clockwise (CW) and counterclockwise (CCW). To avoid confusion, mathematicians usually use CCW as the “positive” direction.



Types of angles:

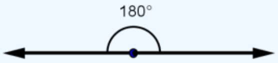
Acute: less than 90° (sharp like a needle).

Right: exactly 90° (corner of a book).



Obtuse: between 90° and 180° (wide and open).

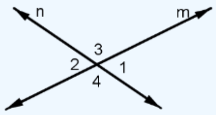
Straight: exactly 180° (a line).



Complementary: two angles adding up to 90°.

Supplementary: two angles adding up to 180°.

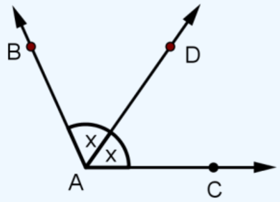
Adjacent: angles sharing a vertex and a side.



Vertical: opposite angles formed when two lines cross; they are always equal.

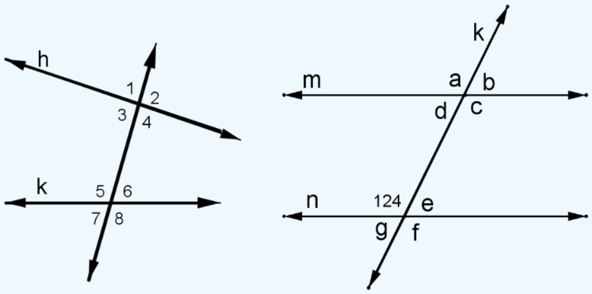
Definition 1.5

An *angle bisector* is a ray that divides an angle into two equal smaller angles. It is the "fair splitter" of angles.



Definition 1.6

Two lines are *parallel* if they never meet, no matter how far they are extended.

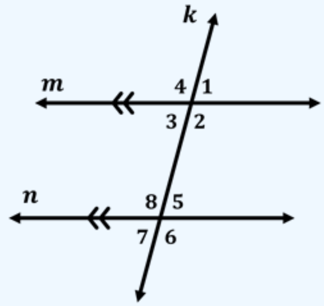


Definition 1.7

A *secant line* cuts across another figure (such as a circle), touching it at two points.

Definition 1.8

When a line (called a *transversal*) crosses parallel lines, many special angles appear:



Corresponding angles: same side, same position. (like $\angle 1\ and\ \angle 5$ or $\angle 2\ and\ \angle 6$)

Alternate interior angles: opposite sides of the transversal, between the parallels. (like $\angle 3\ and\ \angle 5$ or $\angle 2\ and\ \angle 8$)

Alternate exterior angles: opposite sides of the transversal, outside the parallels. (like $\angle 1\ and\ \angle 7$ or $\angle 4\ and\ \angle 6$)

Consecutive interior angles: same side of transversal, between the parallels. (like $\angle 2\ and\ \angle 5$ or $\angle 3\ and\ \angle 8$)

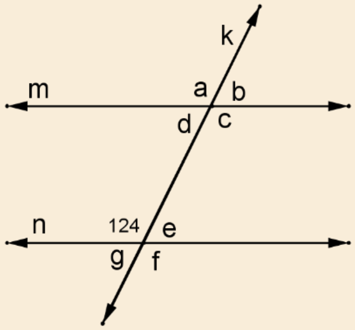
Theorem 1: If two lines are parallel, then corresponding angles are equal.

Theorem 2: If corresponding angles are equal, then the lines are parallel.

Example

On the provided figure, lines m, n are parallel. And k is a traversal line. We have a measure of one angle as shown. Find the measure of the angles:

$$\angle a, \angle b, \angle c, \angle d, \angle e, \angle f, \angle g$$



Solution:

Since m, n are parallel, we have the following measures:

$\angle a = 124°$ corresponding angle

$\angle b = 56°$ supplementary to $\angle a$

$\angle c = 124°$ vertical angle to $\angle a$

$\angle d = 56°$ supplementary to $\angle a$

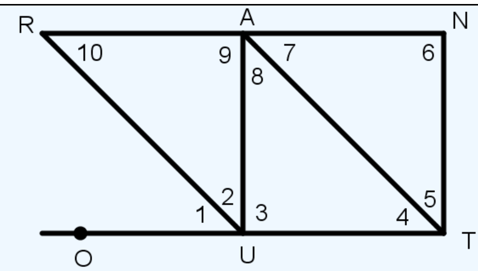
$\angle e = 12$ corresponding to angle $\angle b$

$\angle f = 124°$ corresponding to angle $\angle c$

$\angle g = 56°$ corresponding to angle $\angle d$

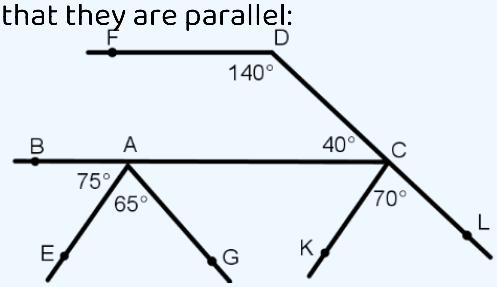
Exercises

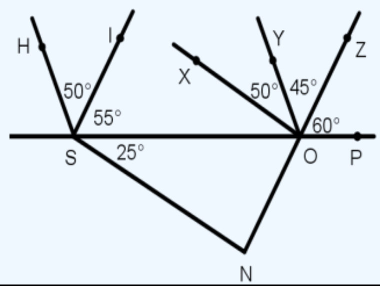
(1) On the figure below, find all pairs of parallel lines in each of the following cases:



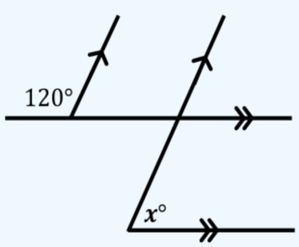
a) $\angle 1 \simeq \angle 4$
b) $\angle 2 \simeq \angle 10$
c) $\angle 5 \simeq \angle 7$
d) $\angle 5 \simeq \angle 8$
e) $\angle 6 \simeq \angle 9 = 90°$
f) $\angle 6 \simeq \angle 3 = 90°$
g) $\angle 7 \simeq \angle 10 \simeq \angle 1$

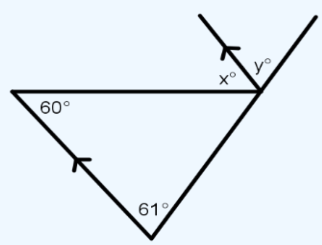
(2) In the following figures, find the parallel lines and determine which angles were used to show that they are parallel:



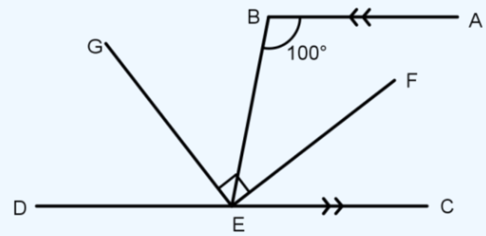


(3) In the following figures, find the measure of the unknown angles:

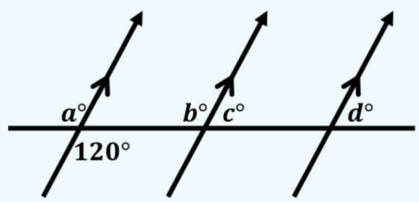




(4) In the following figure, we have $AB \parallel DC$. And EF bisects $\angle BEC$. Find the measure of $\angle BEG, \angle DEG$.

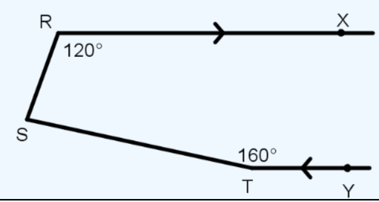


$(\mathbf{5})$ In the following figure, Find the measure of $\angle d$.

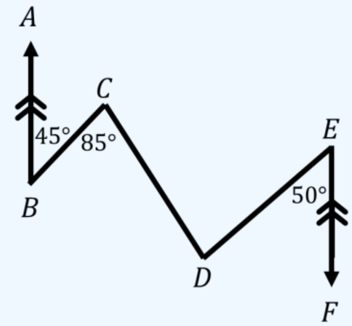


(6) In the following figure, Find the measure of $\angle RST$.

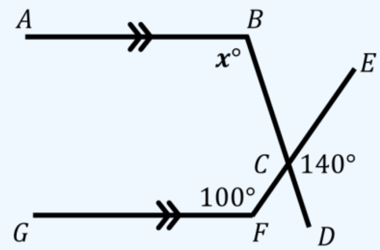
(Hint, draw a line passing through S and parallel to lines $\overline{RX}, \overline{TY}$.)



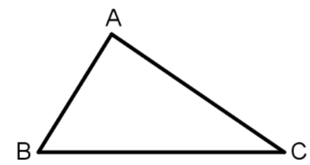
$(\mathbf{7})$ In the following figure, we have $\overrightarrow{BA} \parallel \overrightarrow{EF}$, find the measure of $\angle CDE$.



$(\mathbf{8})$ In the following figures, find the measure of $\angle x$:

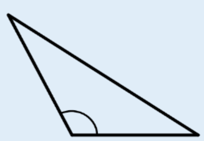
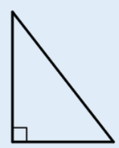
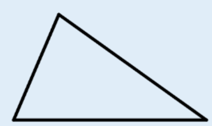
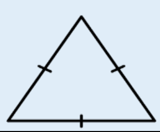
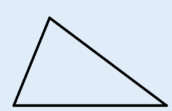


2-Triangles



Triangles are one of the core shapes in geometry and is a special case of polygons.

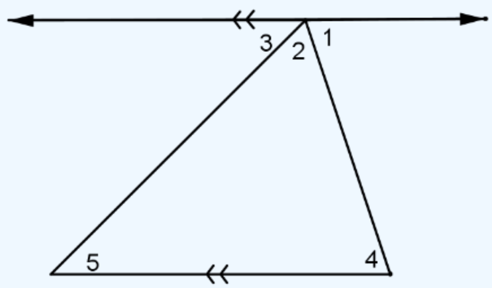
The symbol △ is usually used to denote a triangle. The triangle is built using three different segments. The three vertices connecting the segments are usually called A, B, C and the three segments are called sides and usually denoted by $\overline{AB}$,$\overline{BC}$,$\overline{CA}$. Finally, the three angles of the triangle are denoted by $\angle A$, $\angle B$, $\angle C$.

Obtuse Triangle	Right Triangle	Acute Triangle
		
one angle greater than 90°	one angle is exactly 90°	all angles less than 90°
	The side opposite to the 90° angle is called the hypotenuse while both of the other sides adjacent to the 90° angle are called bases/legs	
Equilateral Triangle	Isosceles Triangle	Scalene Triangle
		
all three sides equal	two equal sides	all three sides different
In this case, all angles are equal to 60°	The two angles opposite to the equal sides are also equal. The two equal sides are called legs while the third side is called the base.	

Theorem:

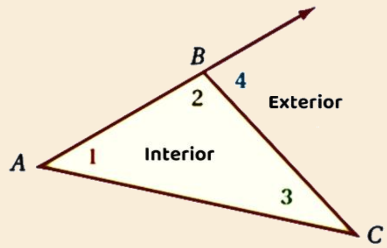
The three interior angles of any triangle add up to $180°$.

Proof. To prove that, draw a line passing through a vertex parallel to the side opposite to that vertex. Since angles $\angle 1, \angle 2, \angle 3$ lie on a straight line, they sum up to $180°$. Finally, notice that $\angle 1 \simeq \angle 4$ and $\angle 3 \simeq \angle 5$ by alternate interior angles. Therefore $\angle 2 + \angle 4 + \angle 5 = 180°$.



From this theorem, many facts follow naturally:

- Extending one side forms an exterior angle, which equals the sum of the two non-adjacent interior angles.
- If two angles are equal, the opposite sides are equal, and vice versa.

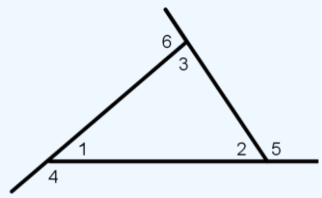


- A triangle can have at most one right or one obtuse angle.
- In a right triangle, the other two angles are acute and together add up to $90°$.

Exercises

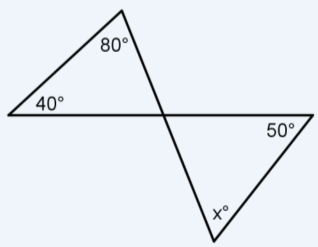
(1) Using the adjacent figure, calculate the following angles:

1. If $\angle 1 = 40°, \angle 2 = 60°$, then find $\angle 6$.
2. If $\angle 1 = 45°, \angle 3 = 70°$, then find $\angle 5$.
3. If $\angle 2 = 50°, \angle 3 = 65°$, then find $\angle 4$.
4. If $\angle 4 = 135°, \angle 2 = 60°$, then find $\angle 3$.
5. If $\angle 5 = 120°, \angle 1 = 40°$, then find $\angle 3$.
6. Find $\angle 4 + \angle 5 + \angle 6$

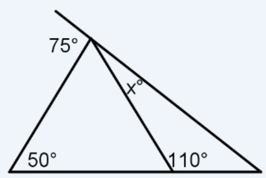


In $(\mathbf{2}-\mathbf{5})$ find the value of $\boldsymbol{x}$ from the given figure:

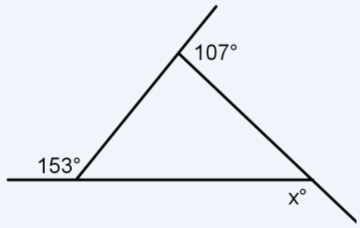
(2)



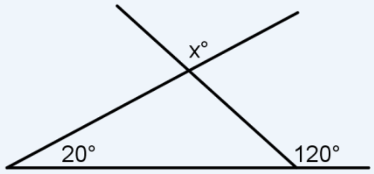
(3)



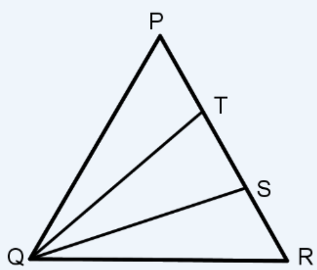
(4)



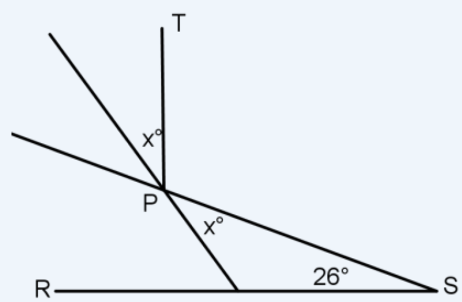
(5)



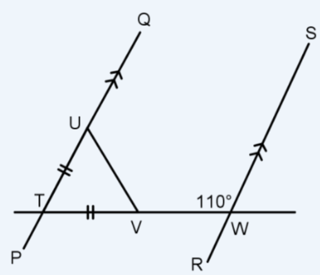
(6) In the adjacent figure: $\triangle QPR$ is an equilateral triangle. QT, QS divides $\angle PQR$ into three equal parts. Find the measure of $\angle QTP$.



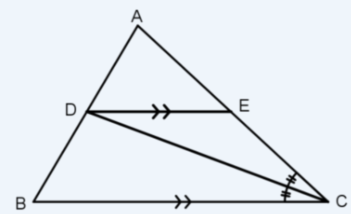
(7) In the adjacent figure: A light ray exits from point S, reflects at point P, and exits in the direction of point T such that RS is perpendicular to PT. Find the value of x.



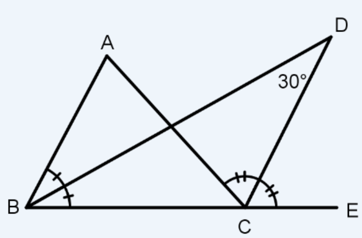
(8) In the adjacent figure, lines $RS \parallel PQ$, and $TU = TV$, and $\angle SWV = 110°$. Find $\angle QUV$.



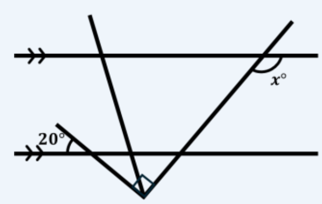
(9) in the adjacent figure: CD bisects $\angle ACB$, and $\angle ACB = 40°$, and $\angle B = 70°$, and $DE \parallel BC$. Find the measures of both of $\angle EDC$, $\angle BDC$.



(10) In the adjacent figure: find the value of x.

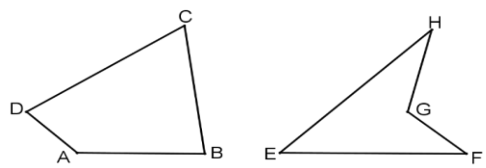


(11) in the adjacent figure: The bisector of $\angle ABC$ and the bisector of $\angle ACE$ intersect at point D. If $\angle BDC = 30°$, find the measure of $\angle A$.



3-Polygons

A *polygon* is a closed figure made by joining straight segments end to end. The corners are called *vertices*, and the sides are the segments themselves. A *diagonal* is a segment joining two non-adjacent vertices.



Definition 3.1

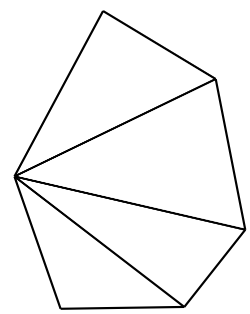
A polygon is *convex* if all its interior angles are less than $180°$. If one of the interior angles is greater than $180°$, it is *concave*.

Thus, a triangle is a 3-sided polygon, a quadrilateral has 4 sides, and so on.

Theorem 3.1

The sum of the interior angles of an n-sided polygon is $(n-2) \times 180°$.

Proof. Choose any of the polygon vertices, and draw all of its diagonals from that vertex.



This divides the polygon into $(n-2)$ triangles. Each triangle has an angle sum of $180°$. So, the total angle sum is:

$$(n-2) \times 180°$$

Definition 3.2

A polygon is *regular* if all its sides and all its angles are equal.

Theorem 3.2

Each interior angle of a regular n-sided polygon measures

$$\frac{(n-2) \times 180°}{n}.$$

Exercises:

(1) Find the sum of interior angles of a polygons with number of sides:

$a)$ 15

$b)$ 16

$c)$ 17

(2) Find the measure of the interior angle of a regular polygon of sides:

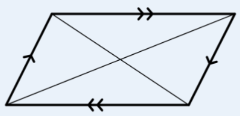
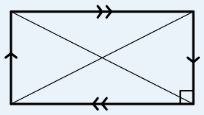
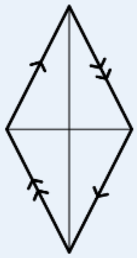
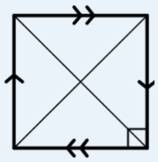
$a)$ 15

$b)$ 16

$c)$ 17

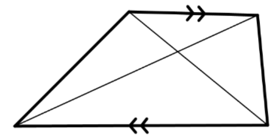
4-Quadrilaterals

Quadrilaterals are polygons with four sides. They come in many "personalities."

	Parallelogram	Rectangle	Rhombus	Square
Shape				
Definition	A quadrilateral with both pairs of opposite sides parallel.	A parallelogram with one right angle; all angles are right angles.	A parallelogram with all sides equal.	A rectangle with all sides equal.
Sides	Opposite sides equal.	Opposite sides equal.	All sides equal.	All sides equal.
Angles	Opposite angles equal.	All angles are right (90°).	Opposite angles equal.	All angles are right (90°).
Diagonals	Bisect each other.	Equal and bisect each other.	Bisect at right angles; vertices-connecting diagonal bisects angles.	Equal, bisect each other, perpendicular, angle-bisecting.
Area	Base × height	Length × width	Base × height	Side^2
Perimeter	Sum of all sides	2 × (length + width)	4 × side	4 × side

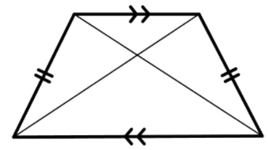
Definition 4.1

A trapezoid (or trapezium) is a quadrilateral with at least one pair of parallel sides.



Definition 4.2

If the non-parallel sides (the *legs*) are equal, the trapezoid is called *isosceles*.

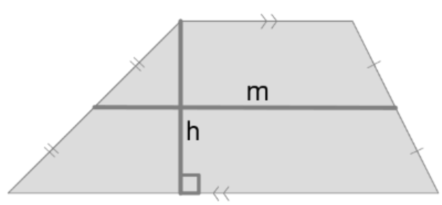


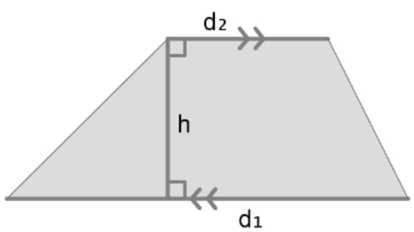
Theorem 4.1

The area of a trapezoid with bases b_1 and b_2, and height h, is

$$Area = \frac{(b_1 + b_2)}{2} \cdot h.$$

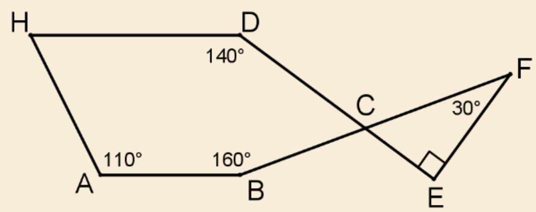
or using the mid-segment : $A = h \cdot m$





Example:

In the figure below: we have two lines DE, BF intersecting at point C, such that EF is perpendicular to DE, and $\angle ABC = 160°$, and $\angle HAB = 110°$. Prove that $AB \parallel HD$.



Solution:

In the right triangle $\triangle FEC$ at $\angle C$, since $\angle CFE = 30°$,

$$\angle FCE = 180° - (30° + 90°) = 60°$$

So by vertical angles, $\angle DCB = 60°$. And since the shape $ABCDH$ is a pentagon, then the sum of its interior angles is:

$$\angle A + \angle B + \angle C + \angle D + \angle H \ 180°(n-2) \ \ 180°(5-2) \ \ 180° \cdot 3 \ \ 540°$$

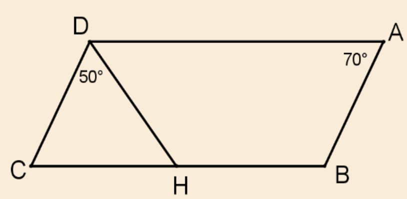
Therefore,

$$\angle H \ 540° - (110° + 160° + 60° + 140°) \ \ 540° - 470° = 70°$$

And since $\angle H + \angle A = 70° + 110° = 180°$ and both are internal angles lying on the same side of the transversal AH, then $AB \parallel HD$.

Example:

In the figure: $ABCD$ is a parallelogram where H lies on side BC, and $\angle A = 70^{\circ}$, $\angle CDH = 50^{\circ}$. Find the measure of $\angle BHD$.



Solution:

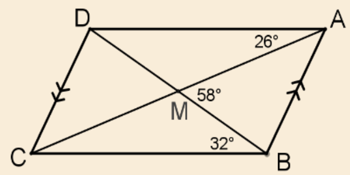
Since $ABCD$ is a parallelogram, then $\angle C = \angle A = 70^{\circ}$ (opposite angles in a parallelogram).

And since $\angle DHB$ lies outside $\triangle DHC$, then

$$\angle DHB \ \angle C + \angle CDH \ \ 50^{\circ} + 70^{\circ} \ \ 120^{\circ}$$

Example:

In the figure: $ABCD$ is a quadrilateral with diagonals intersecting at M, and $AB \parallel CD$, $\angle AMB = 58^\circ$, $\angle MBC = 32^\circ$.
Prove that $ABCD$ is a parallelogram.



Solution:

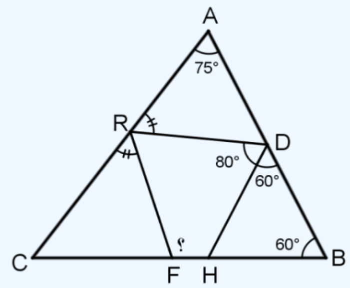
Since $\angle AMB$ lies outside triangle $\triangle CMB$, then $\angle AMB = \angle MBC + \angle MCB$. Therefore,

$$58^\circ = 32^\circ + \angle MCB \Longrightarrow \angle MCB = 26^\circ$$

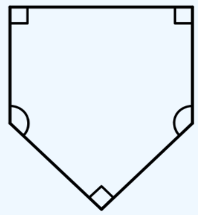
and $\angle CAD, \angle MCB$ are angles in an alternate position, so $AD \parallel BC$ and we already have from the given that $AB \parallel CD$ which implies $ABCD$ is a parallelogram.

Exercises

(1) In the figure: $\triangle ABC$, where $\angle BDH = \angle ABC = 60^{\circ}$ and $\angle HDR = 80^{\circ} = \angle BAC = 75^{\circ}$. Find the measure of $\angle HFR$.



(2) In American baseball, the base is shaped as a pentagon (as shown in the adjacent figure) consisting of three right angles and two identical angles. Find the measure of these two angles.

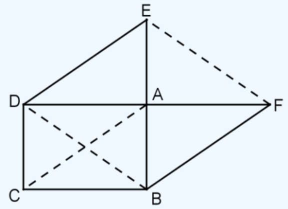


(3) The surface areas of a rhombus and an isosceles trapezoid are equal. If the side length of the rhombus is 10 and the length of the mid-base of the trapezoid is 15, find the ratio between their heights.

Note: The mid-base of a trapezoid is the segment connecting the midpoints of the non-parallel sides.

(4) A trapezoid with one of its parallel bases equal to 3 times the length of the other. If its height equals the length of its mid-base and its area is 100, find the lengths of the two parallel bases.

(5) In the figure below: $ABCD$ is a rectangle, and $ACBF$, $ACDE$ are parallelograms. Prove that $EF \parallel BD$.



Third Unit: Number Theory

Divisibility and Prime Factorization

Summary: This file is an introductory guide to the basic concepts of integer divisibility and prime factorization. We will review a set of practical divisibility rules, define prime and composite numbers, and establish the most important theorem in number theory: The Fundamental Theorem of Arithmetic. Finally, we will apply these concepts to show how factorization is a powerful tool for simplifying complex mathematical expressions.

1- Divisibility Rules

Before diving into factorization, it's useful to have a set of rules to quickly determine if a number is divisible by another. Formally, we say that an integer a divides an integer b, written as $a|b$, if there is an integer k such that $b=ak$.

- **Divisibility by 2:** A number is divisible by 2 if its last digit is even (0, 2, 4, 6, or 8).
  Example: The number 538 is divisible by 2 because its last digit is 8.
- **Divisibility by 3:** A number is divisible by 3 if the sum of its digits is divisible by 3.
  Example: The number 741 is divisible by 3 because the sum of its digits is 12 (7+4+1=12), and 12 is divisible by 3.
- **Divisibility by 4:** A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

  Example: The number 1,824 is divisible by 4 because the number 24 is divisible by 4.

- **Divisibility by 5:** A number is divisible by 5 if its last digit is 0 or 5.

  Example: The number 9,875 is divisible by 5 because its last digit is 5.

- **Divisibility by 6:** A number is divisible by 6 if it is divisible by both 2 and 3.

  Example: The number 432 is divisible by 6 because it is divisible by 2 (last digit is 2) and by 3 (sum of digits is 9).

- **Divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.

  Example: The number 2,853 is divisible by 9 because the sum of its digits is 18, and 18 is divisible by 9.

- **Divisibility by 10:** A number is divisible by 10 if its last digit is 0.

  Example: The number 12,340 is divisible by 10.

- **Divisibility by 11:** A number is divisible by 11 if the alternating sum (odd-placed digits minus even-placed digits) of its digits is divisible by 11.

  Example: Let's take the number 54,384. The alternating sum is $5-4+3-8+4=0$. Since 0 is divisible by 11, the number 54,384 is divisible by 11.

Example question: You have three numbers. Each one is divisible by one of the following numbers: 7, 9, or 11. Match each number with its correct divisor. The numbers are:

- 819,045
- 792,143
- 16,926

Solution: To solve this problem, we will apply the divisibility rules to each number to identify the correct divisor.

1. Testing the number 819,045, we will start by testing for divisibility by 9 because it is the easiest. Rule for divisibility by 9: A number is divisible by 9 if the sum of its digits is divisible by 9. Sum of the digits of 819,045:

$$8 + 1 + 9 + 0 + 4 + 5 = 27.$$

Since 27 is divisible by 9, the number 819,045 is divisible by 9. Therefore, the number 819,045 is divisible by 9.

2. Testing the number 792,143. This number looks complicated, so the divisibility rule for 11 is a good candidate to test. Rule for divisibility by 11: A number is divisible by 11 if the alternating sum of its digits is divisible by 11. Application: Let's calculate the alternating sum of the digits of 792,143 (starting from the right with subtraction, then addition): 3−4+1−2+9−7=−1+1−2+9−7=0−2+9−7=−2+2=0. Result: Since 0 is divisible by 11, the number 792,143 is divisible by 11. Therefore, the number 792,143 is divisible by 11.
3. The number 16,926, since we have found the numbers divisible by 9 and 11, it is logical that this number must be the one divisible by 7.

2- Prime Numbers

All integers greater than 1 are either prime numbers or are composite, made up of a product of prime numbers.

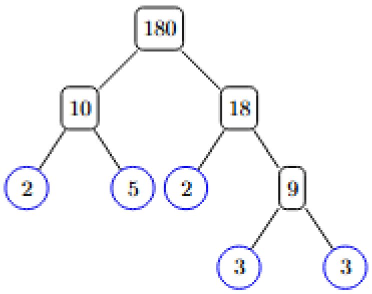
- **Definition (Prime Number):** A prime number is an integer greater than 1 whose only positive divisors are 1 and itself. Examples of prime numbers: (2, 3, 5, 7, 11, 47, 97).
- **Definition (Composite Number):** A composite number is an integer greater than 1 that is not prime. Examples of composite numbers: (4, 9, 15, 27, 180, 588).

The Fundamental Theorem of Arithmetic: Every integer greater than 1 is either a prime number or can be represented as a product of prime numbers, and this representation is unique, except for the order of the factors.

3- Methods of Prime Factorization

Factor Tree Method Example (Factoring the number 180):

- We break down the number 180 until we reach its prime factors. Note that 180 is the product of 10 and 18.
- We break down 18 into 9 and 2, and 10 into 2 and 5.
- We break down 9 into 3 and 3.
- Thus, the factorization of 180 is the product of the prime numbers $2^2 \times 3^2 \times 5$.



Repeated Division Method Example (Factoring the number 588):

We divide the number by the smallest prime that divides it until we reach 1. Then we count the divisors.

<table>
<tr>
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$$588 \div 2 = 294$$
$$294 \div 2 = 147$$
$$147 \div 3 = 49$$
$$49 \div 7 = 7$$
$$7 \div 7 = 1$$

So,

$$588 = 2^2 \times 3 \times 7^2$$

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It is sometimes easier to look at it in this way (*column factorization*)):

<table>
<tr><td>2</td><td>588</td></tr>
<tr><td>2</td><td>294</td></tr>
<tr><td>3</td><td>147</td></tr>
<tr><td>7</td><td>49</td></tr>
<tr><td>7</td><td>7</td></tr>
<tr><td></td><td>1</td></tr>
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4-How Factorization Simplifies Calculations

The true power of factorization lies in its ability to reveal the internal structure of numbers, enabling us to simplify seemingly complex arithmetic operations like division, addition, and subtraction.

- **Simplifying Division (Fractions)** When dividing two large numbers (or simplifying a fraction), finding their common factors is the ideal method.

Example (Simplify the fraction):

$$\frac{396}{924}$$

We factor the numerator and the denominator using one of the methods to get:

$$396 = 2^2 \times 3^2 \times 11, \qquad 924 = 2^2 \times 3 \times 7 \times 11$$

So, when dividing the two numbers, we can cancel out the common prime factors to get:

$$\frac{396}{924} = \frac{2^2 \times 3^2 \times 11}{2^2 \times 3 \times 7 \times 11} = \frac{2^2}{2^2} \times \frac{3^2}{3} \times \frac{1}{7} \times \frac{11}{11} = \frac{3}{7}$$

Exercises:

(1) What is the result of the calculation $(20+18) \div (20-18)$?

$(A) 18$ $(B) 19$ $(C) 20$ $(D) 34$ $(E)\ 36$

(2) Which of the following numbers is closest to the result of the operation $\frac{17 \times 0.3 \times 20.16}{999}$?

$(A)\ 0.01$ $(B)\ 0.1$ $(C)\ 1$ $(D)\ 10$ $(E)\ 100$

(3) Mustafa knows that 1111×1111=1234321. What result does he get when calculating

$(A) 3456543$ $(B) 2345432$ $(C) 2234322$ $(D) 2468642$ $(E)\ 4321234$

(4) What number must replace the star (⋆) to make the following equation true: 2·18·14=6·⋆·7?

$(A) 8$ $(B) 9$ $(C) 10$ $(D) 12$ $(E)\ 15$

(5) Which of the following statements is correct?

$(A) \frac{4}{1} = 1.4$ $(B) \frac{5}{2} = 2.5$ $(C) \frac{6}{3} = 3.6$ $(D) \frac{7}{4} = 4.7$ $(E) \frac{8}{5} = 5.8$

(6) Which of the following fractions is smaller than 2?

$(A) \frac{19}{8}$ $(B) \frac{20}{9}$ $(C) \frac{21}{10}$ $(D) \frac{22}{11}$ $(E) \frac{23}{12}$

(7) 2016 hours is how many weeks?

$(A) 6$ $(B) 8$ $(C) 10$ $(D) 12$ $(E)\ 16$

(8) Hamza has 20 Riyals, and each of his four brothers has 10 Riyals. How much must Hamza give to each brother so that everyone has the same amount of money?

$(A) 2 \quad (B) 4 \quad (C) 5 \quad (D)\ 8 \quad (E)\ 10$

(9) One-sixth of the audience in a children's theater are adults, the rest are children. Two-fifths ($\frac{2}{5}$) of the children are girls. What fraction of the total audience are boys?

$(A)\frac{1}{2} \quad (B)\frac{1}{3} \quad (C)\frac{1}{4} \quad (D)\frac{1}{5} \quad (E)\frac{2}{5}$

(10) Four cousins are 3, 8, 12, and 14 years old. Fatima is younger than Kholoud. The sum of Salwa's and Fatima's ages is divisible by 5, and so is the sum of Salwa's and Kholoud's ages. What is the age of Inas (the fourth cousin)?

$(A) 14 \quad (B) 12 \quad (C) 8 \quad (D) 3 \quad (E) 1448$

(11) If you multiply the three digits of a three-digit number, you get 135. What result do you get by adding these three digits?

$(A) 14 \quad (B) 15 \quad (C) 16 \quad (D)\ 17 \quad (E)\ 18$

(12) A rectangle has an area of 12 cm^2. The lengths of its sides are natural numbers. Which perimeter could the rectangle have?

$(A) 20\ cm \quad (B) 26\ cm \quad (C) 28\ cm \quad (D) 32\ cm \quad (E)\ 48\ cm$

(13) Ahmed writes three single-digit numbers on the board. Saud adds them and gets 15. Then he deletes one of the three numbers and replaces it with the number 3. Ahmed multiplies these three new numbers and gets 36. What are the numbers that Saud could have deleted?

$(A) 6\ or\ 7 \quad (B) 7\ or\ 8 \quad (C)\ 6 \quad (D)\ 7 \quad (E)\ 8$

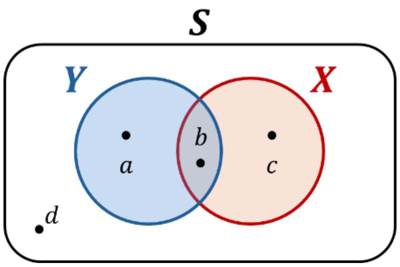
Forth Unit: Combinatorics

The Science of counting. Combinatorics

First: Counting using Venn forms

Venn forms are used to solve counting problems involving overlapping types in adjectives.

Types are usually represented by overlapping circles, each of which has a number representing the number of elements of that type as shown in the figure:



a Achieves only the property Y

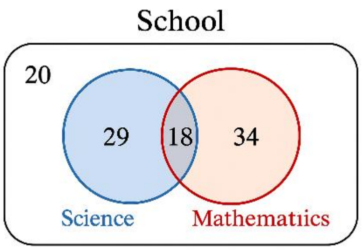
c Achieves only the property X

b Achieves both properties X and Y together

d does not achieve the property X and does not achieve the property Y (does not achieve nor property X or Y)

Example:

The following figure represents the numbers of students in a school who prefer a math class and who prefer a science class.



a) How many students prefer math? ....

b) How many students prefer science or math? ....

c) How many students prefer science and math together? ....

d) How many students prefer science only? ....

e) How many students do not prefer science? ....

f) How many students does the school have? ....

Solution

a) $18 + 34 = 52$
b) $29 + 18 + 34 = 81$
c) 18
d) 29
e) $18 + 34 + 20 = 72$
f) $18 + 34 + 29 + 20 = 101$

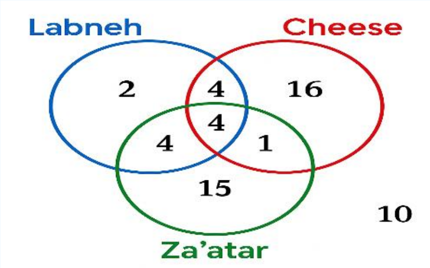
Exercises

(1) In one of our gifted classes, 35 students were offered to learn a foreign language, namely English and French. 15 students Choose an English language, and 5 students choose both languages. If we knew that any student chose to learn at least one of the two languages, how many students chose to learn only French?

(2) 40 People went on a trip, 18 of them prefer cheese pie and 15 of them prefer labneh pie, while 12 of them don't prefer either cheese or labneh. How many students prefer the two types together?

(3) A poll was conducted for a 50 students at a school to find out what type of sport they prefer, and the result was that 33 of them prefer soccer, 24 prefer basketball, and 11 prefer both. How many students don't prefer either two games?

(4) The figure shows the distribution of 60 students in a school, according to their preference for three types of pies. How many students prefer labneh?



(5) A factory manager conducted liked all three products from the factory, and found that 38 persons liked product A, 36 liked product B, 39 liked product C, 24 liked both products A and B, 20 liked both products C and A, 18 liked both products B and C, and finally 9 people liked all three products. How many people have liked only product C?

Second: Counting a list of numbers

It is easy to count numbers in the list $1,2,3,4,\ldots,50$ and therefore it is called a simple list, hence it can be said that the list of numbers is simple if you meet the following conditions:

1- Starts with the number 1   2- has Consecutive contiguous numbers

Example:

How many numbers are in the list $3,6,9,\ldots,327$

Solution:

The list is not simple, but it can be converted into a simple list by dividing all the numbers in the list by 3

$\div 3$	$3,6,9,\ldots,327$	
	$1,2,3,\ldots,109$	Simple list

so, the number of numbers is equal to 109

Example: How many numbers are in the list $23,28,33,\ldots,548$

Solution: The list is not simple but can be converted into a simple list with the following steps

-3	$23,28,33,\ldots,548$	
$\div 5$	$20,25,30,\ldots,545$	
-3	$4,5,6,\ldots,109$	
	$1,2,3,\ldots,106$	Simple list

So, the number of numbers is equal to 106

Some lists can be converted to a simple list by performing an operation on all numbers in them, because operations on all numbers produce a list with the same number of numbers.

And we will see how to do that in the following exercises

Exercises

(6) How many numbers are in the following list $1,2,3,\dots,1440$?

(7) How many numbers are in the following list $8,9,\dots,2019$?

(8) How many numbers are in the following list $5,9,13,\dots,505$?

(9) How many positive even numbers are less than 2025 ?

(10) How many numbers are in the following list: $\frac{3}{7},1,\frac{11}{7},\dots,289$?

(11) How many numbers are there between 2023 , 63 so that it is a multiple of the number 3 ?

(12) How many positive integers are less than 600 so that it is a whole square?

(13) How many positive integers are less than 1447 so that it is multiple of both numbers 3 and 5?

(14) How many positive integers are less than 2025 which are multiples of the number 7 and not multiples of the number 5?

(15) How many positive integers are less than 500 which are multiples of the number 7 and not even?

Solutions

Integers and Their Properties:

Exercises:

(1)

$a)\ (-4) + 9 = 5$
$b)\ -42 \div 7 = -6$
$c)\ (2)^5 = 32$
$d)\ (-4)^3 = -64$
$e)\ (-5)^2 = 25$
$f)\ |0| = 0$
$g)\ (-4) \times (-8) = 32$
$h)\ -8 - (-5) = -8 + 5 = -3$
$i)\ -1 - 4 + 7 = 2$
$j)\ 2 \times 4 + 6 \times 5 = 8 + 30 = 38$
$k)\ |-6| = 6$
$m)|-3| - |-7| = 3 - 7 = -4$

(2)

$a)\ a^2 \times a^5 = a^{2+5} = a^7$
$b)\ x^7 \div x^3 = x^{7-3} = x^4$
$c)\ (a^3)^4 = a^{3\times4} = a^{12}$
$d)\ (x^2)^3 \times (x^4)^5 = x^{2\times3} \times x^{4\times5} = x^6 \times x^{20} = x^{26}$
$e)\ \dfrac{a^3 \times a^7}{a^2 \times a^6} = \dfrac{a^{3+7}}{a^{2+6}} = \dfrac{a^{10}}{a^8} = a^{10-8} = a^2$
$f)\ \dfrac{a^4 \times a^5}{a^3 \times a^6} = \dfrac{a^{4+5}}{a^{3+6}} = \dfrac{a^9}{a^9} = a^{9-9} = a^0 = 1$

(3)

$a)\ (-7) + (-12) - (-14) - (-15) - (-18) - (-38)$
$= -7 - 12 + 14 + 15 + 18 + 38$
$= 66$
$b)\ (-5)^2 + |-6| - (-1)^{1447}$
$= 25 + 6 + 1$
$= 32$

Challenge Problems:

(1)

$$-1-(-1)^1-(-1)^2-(-1)^3-\cdots-(-1)^{99}-(-1)^{100}$$
$$=-1+\underbrace{(1-1+1-1+\cdots+1-1)}_{100\ terms}$$
$$=-1+0$$
$$=-1$$

(2)

$$1234\times 9999$$
$$=1234(10000-1)$$
$$=1234\times 10000-1234\times 1$$
$$=12340000-1234$$
$$12338766$$

(3)

$$5\{(2a-3)-[7(4a-1)-20]\}-(3-8a)$$
$$=5\{2a-3-28a+7+20\}-3+8a$$
$$=5\{-26a+24\}-3+8a$$
$$=-130a+120-3+8a$$
$$=-122a+117$$

(4)

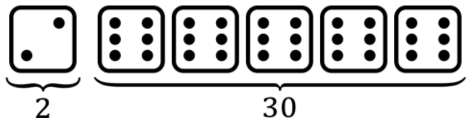
The number of orbits completed by a satellite in one week equals the number of hours in a week divided by 7.

$$=(24\times 7)\div 7=\frac{24\times 7}{7}=24$$

(5)

To obtain the smallest number on one of the dice, the other five dice must show the largest possible number.

Therefore, this occurs when each of the other five dice shows the number 6, and then the smallest number appears on the sixth die, which is 2.



(6)

$$1-2+3-4+\cdots-100+101$$
$$=(1-2)+(3-4)+\cdots+(99-100)+101$$
$$=\underbrace{-1-1-1\ldots-1}_{50\ times}+101$$
$$=-50+101$$
$$=51$$

Rational Numbers $\mathbb{Q}$

Exercises:

(1)

$a)\ \frac{2}{5}+\frac{1}{6}=\frac{12+5}{30}=\frac{17}{30}$

$b)\ \frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\frac{1}{6}$

$c)\ \frac{-3}{8}\times\frac{4}{9}=\frac{-3}{9}\times\frac{4}{8}=\frac{-1}{3}\times\frac{1}{2}=\frac{-1}{6}$

$d)\ \frac{-3}{5}-\left(-\frac{1}{2}\right)=\frac{-3}{5}+\frac{1}{2}=\frac{-6+5}{10}=\frac{-1}{10}$

$e)\ 1\div 3\div 4\div 5=1\times\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}=\frac{1}{60}$

$f)\ \frac{1}{9}-\frac{1}{10}=\frac{10-9}{90}=\frac{1}{90}$

$g)\ (-3)\div 4\times 6\div(-5)=-3\times\frac{1}{4}\times 6\times\left(-\frac{1}{5}\right)=\frac{18}{20}=\frac{9}{10}$

$h)\ \frac{-6}{35}\div\frac{2}{7}=\frac{-6}{35}\times\frac{7}{2}=\frac{-6}{2}\times\frac{7}{35}=\frac{-3}{1}\times\frac{1}{5}=\frac{-3}{5}$

$i)\ \left(\frac{-1}{2}\right)^5=\frac{(-1)^5}{2^5}=\frac{-1}{32}$

(2)

$a)\ 0.\overline{2}=\frac{2}{9}$

$b)\ 0.\overline{37}=\frac{37}{99}$

$c)\ 1.8\overline{23}=\frac{18}{10}+\frac{1}{10}\times 0.\overline{23}$

$$=\frac{18}{10}+\frac{1}{10}\times\frac{23}{99}$$

$$=\frac{18}{10}+\frac{23}{990}$$

$$=\frac{1805}{990}$$

$$=\frac{361}{198}$$

(3)

Yes, the claim is correct.

When we divide the number $\frac{a}{b}$ by $\frac{c}{d}$, the result is equal to $\frac{a}{b} \times \frac{d}{c}$,

which means we multiply by the multiplicative inverse of $\frac{c}{d}$.

(4)

Saleh's claim is correct. Let's take any two rational numbers, for example:

$$\frac{2}{7}, \frac{3}{7}$$

They may look close to each other, and it might seem that there are no rational numbers between them.

However, this is not true. If we rewrite them in an equivalent form, for instance:

$$\frac{20}{70}, \frac{30}{70}$$

we can clearly find numbers between them, such as:

$$\frac{21}{70}, \frac{22}{70}, \dots, \frac{29}{70}$$

And if we express them again with larger equivalent denominators, for example:

$$\frac{200}{700}, \frac{300}{700}$$

We will find even more numbers between them.

Therefore, there are infinitely many rational numbers between any two rational numbers.

(5)

$$-\frac{1}{3}, \ -0.3, \ -0.23$$

Challenge Problems:

(1)

To begin, notice that:

$$\frac{1}{6} = \frac{5}{30} \text{ and } \frac{1}{3} = \frac{10}{30}$$

Therefore, any fraction between them can be written in the form:

$$\frac{k}{30}$$

where k is an even number.

If we let k take the values 6 and 8, we obtain only two fractions.

(2)

We know that

$$\frac{1}{4} = 0.25$$

Therefore, N is equal to one-fourth of 8, and hence

$$N = 2$$

Thus, Khaled will obtain

$$16$$

(3)

In total, Haitham has completed the first half of the journey, plus $\frac{3}{5}$ of the second half.

Therefore, the total distance he has covered is:

$$\frac{1}{2} + \frac{3}{5} \times \frac{1}{2} = \frac{1}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$$

So, Haitham has covered $\frac{4}{5}$ of his journey.

(4)

$$\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{9}-\frac{1}{10}\right)$$
$$=1+\left(-\frac{1}{2}+\frac{1}{2}\right)+\left(-\frac{1}{3}+\frac{1}{3}\right)+\cdots+\left(-\frac{1}{9}+\frac{1}{9}\right)-\frac{1}{10}$$
$$=1-\frac{1}{10}$$
$$=\frac{9}{10}$$

(5)

$$\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdots\cdots\frac{79}{80}$$
$$=\frac{1}{80}$$

(6)

A		7						e	d	c	7	b	4

Since the sum of $\boldsymbol{b}$, $\mathbf{7}$, and 4equals $\mathbf{20}$, it follows that $\boldsymbol{b}=\mathbf{9}$.

Next, $\mathbf{7}+\mathbf{9}+\boldsymbol{c}=\mathbf{20}$, so $\boldsymbol{c}=\mathbf{4}$.

Continuing this pattern to the left, we find $\boldsymbol{d}=\mathbf{9}$ and $\boldsymbol{e}=\mathbf{7}$.

By maintaining this sequence, we finally reach $\boldsymbol{A}=\mathbf{9}$.

(7)

Leila is now 32 years old, and the sum of her two daughters' ages is 5 years.

This means that the difference between Leila's age and the sum of her daughters' ages is 27 years.

Each year, Leila's age increases by one year, while the sum of her daughters' ages increases by two years, and the difference between them decreases by one year.

Since the difference is now 27 years, it will take 27 years for the difference to become zero.

At that time, Leila's age will be $\mathbf{32}+\mathbf{27}=\mathbf{59}$ years.

(8)

The maximum number of matches that any team can play is 2, since each team can play at most once per year, and there are only three teams.

Therefore, Team B and Team C must each have played only one match.

As for Team A, it must have played two matches, because if it had played zero matches, it would have scored zero goals, while according to the problem statement, it has scored either 5 or 3 goals.

Since each recorded number must either increase or decrease by 1, it follows that Team A has:

1 win, 1 draw, and 0 losses (since the total number of matches played is 2).

Because Team A won one match, Team B must have lost one match (since Team B originally had 0 losses, and Team C cannot have lost 2 matches).

Therefore, Team B has 1 loss and 0 wins or draws.

We can now deduce that Team C drew with Team A; thus, Team C has 1 draw, 0 wins and losses.

Since Team C's only match ended in a draw, its goals for and goals against must be equal.

Hence, Team C scored 2 goals and conceded 2 goals.

The only match played by Team B ended in a loss, so the number of goals for Team B must be less than the number of goals against.

This is satisfied when Team B scored 1 goal and conceded 3 goals.

Since both Team B and Team C played only against Team A, the total number of goals scored by Team A equals the sum of the goals conceded by Teams B and C.

Therefore, Team A scored a total of 5 goals.

Similarly, the total number of goals conceded by Team A equals the sum of the goals scored by Teams B and C.

Thus, Team A conceded 3 goals.

Hence, the correct table is as follows:

Team	Played	Won	Drawn	Lost	Goals For	Goals Against
A	2	1	1	0	5	3
B	1	0	0	1	1	3
C	1	0	1	0	2	2

Geometry Solutions

Points, Lines, and Angles

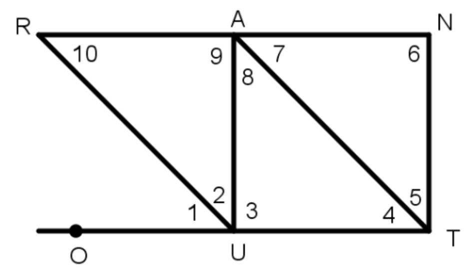
Exercises

(1)

$a)\ \angle 1 \cong \angle 4 \Longrightarrow RU||AT.$

$b)\ m\angle 2 \cong m\angle 10 \Longrightarrow$ no parallel lines needed

$d)\ \angle 5 \cong \angle 7 \Longrightarrow$ no parallel lines needed



$d)\ \angle 5 \cong \angle 8 \Longrightarrow AU||NT.$

$e)\ m\angle 6 = m\angle 9 = 90° \Longrightarrow AU||NT.$

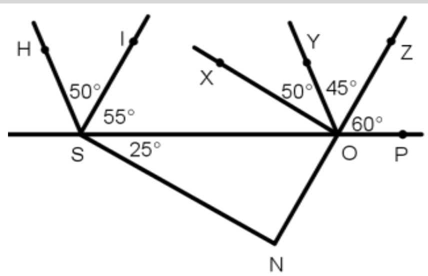
$f)\ m\angle 6 = m\angle 3 = 90° \Longrightarrow$ no parallel lines needed

$g)\ m\angle 7 = m\angle 10 = m\angle 1 \Longrightarrow RU||AT\ , RA||OU$

(2)

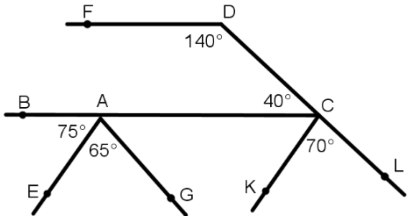
$\angle HSO = \angle YOP = 105 \Longrightarrow HS \parallel YO$

$$\angle XOS = 180 - (60 + 45 + 50) = 25 = \angle NSO \Longrightarrow NS \parallel XO$$



$\angle FDC + \angle DCA = 180 \Longrightarrow FD \parallel AC$

$$\angle CAG = 180 - (75 + 65) = 40 = \angle DCA \Longrightarrow DC \parallel AG$$



(3)

a) $x = 180 - 120 = 60°$

b) $x = 60°$, $y = 61°$

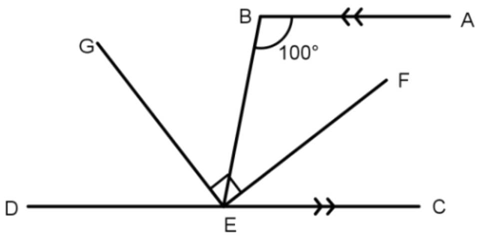
(4)

$\angle BEC = 180 - 100 = 80°$

$\angle BEF = \angle CEF = 80 \div 2 = 40°$

$\angle BEG = 90 - 40 = 50°$

$\angle DEG = 180 - (90 + 40) = 50°$



(5)

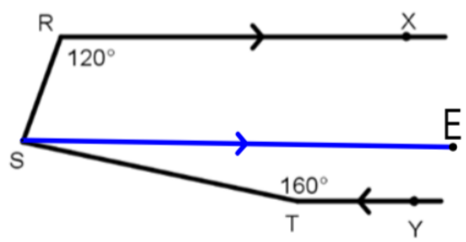
$\angle d = 180 - \angle a = 180 - 120 = 60°$

(6)

$\angle RSE = 180 - 120 = 60°$

$\angle TSE = 180 - 160 = 20°$

$\angle TSR = 60 + 20 = 80°$

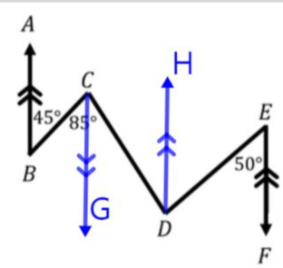


(7)

$\angle BCG = 45°, \angle DCG = 85 - 45 = 40°$

$\angle CDH = 40°, \angle EDH = 50°$

$\angle CDE = 50 + 40 = 90°$

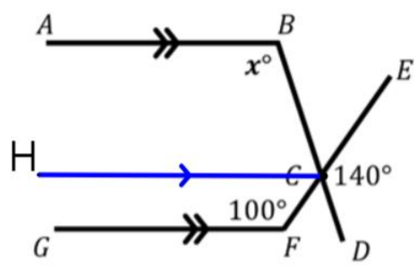


(**8**)

$\angle HCF = 180 - 100 = 80°$

$\angle HCB = 140 - 80 = 60°$

$x = 180 - 60 = 120°$



Triangles:

Exercises:

(1)

$a)\ m\angle 6 = 40 + 60 = 100°$

$b)\ m\angle 5 = 45 + 70 = 115°$

$c)\ m\angle 4 = 50 + 65 = 115°$

$d)\ m\angle 3 = 135 - 60 = 75°$

$e)\ m\angle 3 = 120 - 40 = 80°$

$f)\ m\angle 4 + m\angle 5 + m\angle 6 = 360°$

(2)

$40 + 80 + \angle A = x + 50 + \angle A \Longrightarrow x = 80 + 40 - 50 = 70°$

(3)

$\angle ABC = 75 - 50 = 25°$

$x = 180 - (110 + 25) = 45°$

(4)

$x = 360 - (107 + 153) = 100°$

(5)

$x = 180 - (20 + 60) = 100°$

(6)

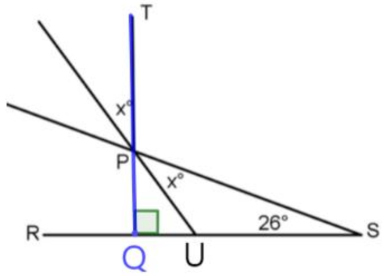
$\angle QPR = 60°\ , \angle PQT = 60 \div 3 = 20°$

$\angle QTP = 180 - (60 + 20) = 100°$

(7)

$\angle QPU = x$

$2x + 26 = 90 \Longrightarrow x = 32°$



(8)

$\angle UTV = 180 - 110 = 70°$

$\angle TVU = \angle TUV = (180 - 70) \div 2 = 55°$

$\angle QUV = 180 - 55 = 125°$

(9)

$\angle EDC = \angle DCB = 40 \div 2 = 20°$

$\angle BDC = 180 - (20 + 70) = 90°$

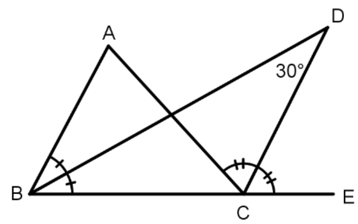
(10)

$\angle ECD = \angle DCA = y$

$\angle ABD = \angle DBC = x$

$y - x = 30$

$\angle A = 2y - 2x = 2(y - x) = 2 \times 30 = 60°$

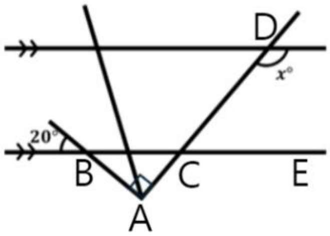


(11)

$\angle ABC = 20°$

$\angle ECD = \angle ACB = 90 - 20 = 70°$

$x = 180 - 70 = 110°$



Quadrilaterals:

Exercises:

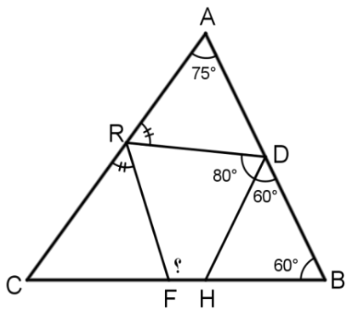
(1)

$\angle FHD = 60 + 60 = 120°$
$\angle ADR = 180 - (60 + 80) = 40°$

$\angle CRF = \angle RDA = 180 - (75 + 40) = 65°$

$\angle FRD = 180 - (65 + 65) = 50°$

$\angle HFR = 360 - (120 + 80 + 50) = 110°$

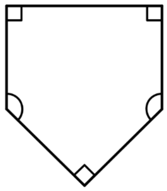


(2)

$3 \times 90° + 2x = (5 - 2) \times 180°$

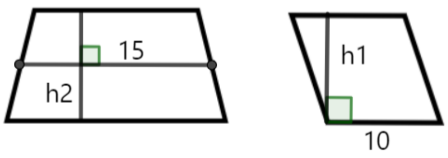
$2x = 270°$

$x = 135°$



(3)

$15 \times h_2 = 10 \times h_1 \Longrightarrow \frac{h_2}{h_1} = \frac{10}{15} = \frac{2}{3}$

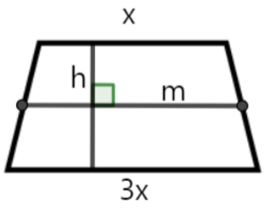


(4)

$h = m = \frac{x + 3x}{2} = 2x$

$A = h \times m = (2x)(2x) = 100$

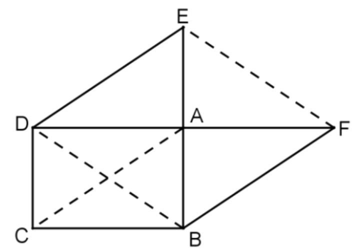
$\Longrightarrow 4x^2 = 100 \Longrightarrow x^2 = 25 \Longrightarrow x = 5 \ , 3x = 15$



(5)

Notice that $DBFE$ is a parallelogram, which means that:

$$BF = CA = DE \Longrightarrow BF = DE\,, BF \parallel CA \parallel DE \Longrightarrow BF \parallel DE$$



Number Theory Solutions

(1)

First, we calculate what is inside the parentheses:

20+18=38, 20−18=2

Next, we perform the division: 38÷2=19.

The correct answer is 19.

(2)

This problem relies on estimation to simplify calculations. We round the numbers to the nearest easy value:

17≈20, 20.16≈20, 999≈1000.

Now the approximate calculation becomes:

$$\frac{20 \times 0.3 \times 20}{1000} = \frac{120}{1000} = 0.12$$

The result 0.12 is closest to 0.1. The correct answer is 0.1.

(3)

We can rewrite the problem using factorization.

Note that 2222=2×1111. So, the operation becomes:

$$1111 \times (2 \times 1111) = (1111 \times 1111) \times 2$$

Since we know the value of (1111×1111) from the question, we substitute it: $1234321 \times 2 = 2468642$

The correct answer is 2468642.

(4)

We use prime factorization to simplify both sides of the equation, just as explained in the lesson.

Left side: $2 \cdot (2 \cdot 3^2) \cdot (2 \cdot 7) = 2^3 \cdot 3^2 \cdot 7$ Right side: $(2 \cdot 3) \cdot \star \cdot 7$

By equating the two sides, we can find the value of the star by cancellation:

$$\star = \frac{2^3 \cdot 3^2 \cdot 7}{2 \cdot 3 \cdot 7} = 2^{3-1} \cdot 3^{2-1} \cdot 7^{1-1} = 2^2 \cdot 3^1 = 12$$

The correct answer is 12.

(5)

We calculate the value of each fraction to check the statement's validity:

(A) $\frac{4}{1} = 1.4$, which is not equal to 1.4.

(B) $\frac{5}{2} = 2.5$. This statement is correct.

(C) $\frac{6}{3} = 3.6$, which is not equal to 3.6.

(D) $\frac{7}{4} = 4.7$, which is not equal to 4.7.

(E) $\frac{8}{5} = 5.8$, which is not equal to 5.8.

The correct answer is 2.5.

(6)

To see if a fraction is smaller than 2, we compare the numerator to twice the denominator.

(A) $19>2\times8=16$.

(B) $20>2\times9=18$.

(C) $21>2\times10=20$.

(D) $22=2\times11=22$.

(E) $23<2\times12=24$. This fraction is smaller than 2.

The correct answer is $\frac{23}{12}$.

(7)

The number of hours in one week is 7 days×24 hours/day=168 hours.

We need to calculate $\frac{2016}{24\times7}$.

We can simplify this fraction using divisibility rules (instead of calculating 24×7).

The correct answer is 12.

(8)

Total amount: 20+(4×10)=60 Riyals.

Number of people: 5.

Equal share: 60÷5=12 Riyals per person.

Each brother needs 12 Riyals. Therefore, Hamza must give 2 Riyals to each brother.

The correct answer is 2.

(9)

Fraction of children = $\frac{5}{6}$.

Fraction of boys = (Fraction of children) * (Fraction of boys among children) = $\frac{3}{5}\times\frac{5}{6}=\frac{1}{2}$.

The correct answer is $\frac{1}{2}$.

(10)

The ages are {3, 8, 12, 14}. Since "Salwa's" age is common in two sums that are multiples of 5, we look for a number that forms a sum divisible by 5 with two other numbers.

3+12=15, and 8+12=20.

Therefore, Salwa's age is 12.

Fatima's and Khaloud's ages are 3 and 8. The ages used are 3, 8, and 12. The remaining age for Inas is 14.

The correct answer is 14.

(11)

We need to find three digits (from 1 to 9) whose product is 135. The best way is the prime factorization of 135.

$$135 = 5 \times 27 = 5 \times 3 \times 9 = 5 \times 3 \times 3 \times 3.$$

The prime factors are {3, 3, 3, 5}.

We must group them to form three digits. One of the numbers cannot be 3×5=15 because it is not a single digit. We can group two factors: 3×3=9. Thus, the three digits are 9 and the two remaining factors 3 and 5. The digits are {3, 5, 9}. Check: 3×5×9=15×9=135. This is correct. Now, we add these digits: 3+5+9=17.

The correct answer is 17.

(12)

Area = Length x Width = 12. Since the sides are natural numbers, we are looking for the factor pairs of 12. The factor pairs of 12 are:

- 1 and 12
- 2 and 6
- 3 and 4

If the dimensions are 1 and 12: Perimeter = 2×(1+12)=2×13=26 cm. If the dimensions are 2 and 6: Perimeter = 2×(2+6)=2×8=16 cm. If the dimensions are 3 and 4: Perimeter = 2×(3+4)=2×7=14 cm. Among the given options, the only possible perimeter is 26 cm.

The correct answer is 26 cm.

(13)

Let the original numbers written by Ahmed be x, y, z.

First given: $x + y + z = 15$. Saud deleted one of the numbers (let's say z) and replaced it with 3. The new numbers are $x, y, 3$.

Second given: $x \cdot y \cdot 3 = 36$. From the second given, we can find x and y from the equation $x \cdot y = 12$. We are looking for two single-digit numbers whose product is 12.

The possible pairs are: 2 and 6, 3 and 4. Now, we use the first given to find the deleted number z.

Case 1: If x=2, y=6. 2+6+z=15⟹8+z=15⟹z=7. The deleted number could be 7. The original numbers were {2, 6, 7}.

Case 2: If x=3, y=4. 3+4+z=15⟹7+z=15⟹z=8. The deleted number could be 8. The original numbers were {3, 4, 8}.

So, the number that Saud deleted could be either 7 or 8.

The correct answer is 7 or 8.

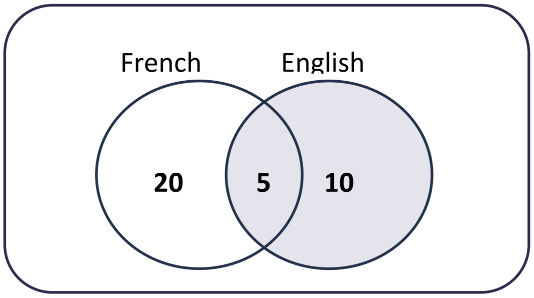
Counting using Venn forms

Exercises:

(1)

Number of people who choose to learn

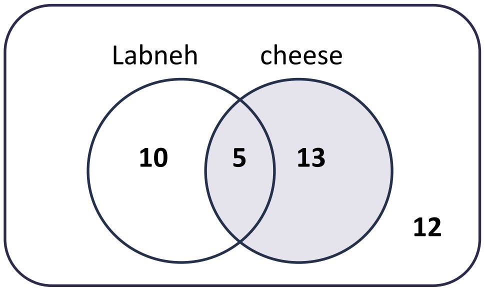
French $=35-15=20$



(2)

Number of people who prefer both types

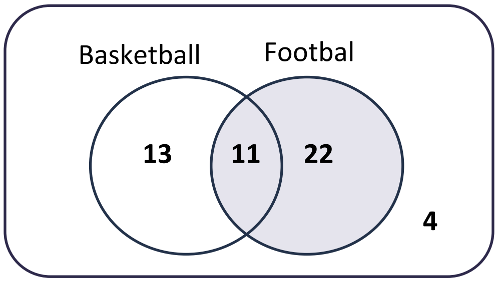
$=12+18+15-40=5$



(3)

Number of people who don't like either game

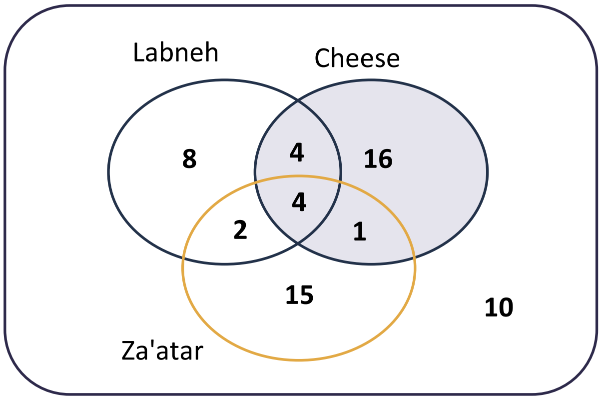
$=12+18+15-40=5$



(4)

Number of people who prefer labneh =

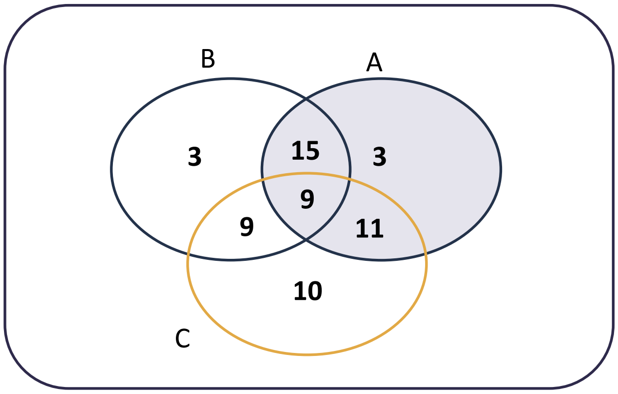
$$60-(16+1+15+10)=18$$



(5)

Number of people who liked product (C) only

$= 39-(11+9+9)=10$



Counting a list of numbers

(6)

A simple list with 1440 numbers

(7)

-7	$8,9,\ldots\ldots,2019$
	$1,2,\ldots\ldots,2012$

A simple list with 2012 numbers

(8)

-1	$5,9,13,\ldots\ldots,505$
$\div 4$	$4,8,12,\ldots\ldots,504$
	$1,2,3,\ldots\ldots,126$

A simple list with 126 numbers

(9)

$\div 2$	$2,4,6,\ldots\ldots,2024$
	$1,2,\ldots\ldots,1012$

A simple list with 1012 numbers

(10)

$*7$	$\frac{3}{7}, 1, \frac{11}{7} \dots \dots, 289$
$+1$	$3, 7, 11, \dots \dots, 2023$
$\div 4$	$4, 8, 12, \dots \dots, 2024$
	$1, 2, 3, \dots \dots, 506$

A simple list with 506 numbers

(11)

-63	$66, 69, \dots \dots, 2022$
$\div 3$	$3, 6, \dots \dots, 1959$
	$1, 2, 3, \dots \dots, 653$

A simple list with 653 numbers

(12)

$$24^2 = 576 < 600 , 25^2 = 625 > 600$$

So, the larger full square is less than 600 is $24^2 = 576$

$\sqrt{\ }$	$1^2, 2^2, \dots \dots, 24^2$
	$1, 2, \dots \dots, 24$

A simple list with 24 numbers

(13)

Common multiples of $\{5,3\}$ are the multiples of 15, And the biggest multiplier of 15 less than 1447 is 1440

$\div 15$	$15,30, \dots \dots .,1440$
	$1,2, \dots \dots .,96$

A simple list with 96 numbers

Solution 2:

$$\left\lfloor \frac{1447}{15} \right\rfloor = 96$$

(14)

multiples of 7

	$7,14, \dots \dots .,2023$
$\div 7$	$1,2, \dots \dots .,289$

L.C.M(5,7) =35

	$35,70, \dots \dots .,1995$
$\div 35$	$1,2, \dots \dots .,57$

So, Number of multiples of 7 and not 5 is $289 - 57 = 232$

Solution 2:

$$\left\lfloor \frac{2025}{7} \right\rfloor - \left\lfloor \frac{2025}{35} \right\rfloor = 232$$

(15)

multiples of 7

$\div 7$	$7,14,\ldots\ldots..,497$
	$1,2,\ldots\ldots..,71$

$L.C.M(2,7) = 14$

$\div 14$	$14,28,\ldots\ldots..,490$
	$1,2,\ldots\ldots..,35$

So, Number of multiples of 7 and not even is $71 - 35 = 36$

Solution 2:

$$\left\lfloor\frac{500}{7}\right\rfloor - \left\lfloor\frac{500}{14}\right\rfloor = 36$$