National Science and Mathematics Olympiad
NSMO

Mathematics 2

Cities and Governorates Competition

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Introduction

Our exceptional sons and daughters,

We are delighted to congratulate you on successfully completing the **Cities and Governorates stage** and qualifying for the **General Administrations stage**—an important, advanced step on your path toward mathematical challenge and innovation.

This **resource packet** is designed to **expand your understanding** across the four main branches of mathematics: **Combinatorics, Geometry, Algebra, and Number Theory**. We will focus on advanced concepts in **counting, geometric visualization, linear equations, and the principles of distribution and multiplication** in Number Theory.

This stage aims to **refine your skills in analytical thinking** and help you **connect mathematical concepts** to one another, applying them effectively in various problem situations.

This resource packet is a valuable opportunity to **deepen your understanding of mathematical patterns** and to use **logical reasoning** for justification and solving problems using organized methods.

We are confident in your abilities and look forward to seeing you **excel** in this crucial phase of your journey toward excellence.

The Scientific Team for the National Science and Mathematics Olympiad (NSMO) – Mathematics Track

First Unit: ALGEBRA

1- Integers and Their Properties

1-1 Sets of Numbers:

Set of Natural Numbers:

$$\boldsymbol{N} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \dots\}$$

Set of Whole Numbers:

$$\boldsymbol{W} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \dots\}$$

Set of Integers:

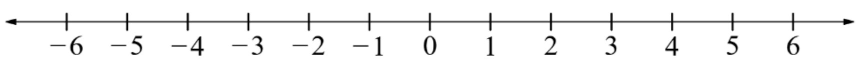
$$\boldsymbol{Z} = \{\dots, -\mathbf{3}, -\mathbf{2}, -\mathbf{1}, \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \dots\}$$

Integers can be divided into three groups:

- Positive integers.
- Zero.
- Negative integers.

Note that the number $+\mathbf{1}$ is usually written simply as $\mathbf{1}$, meaning that the positive sign is often omitted and not pronounced. On the other hand, the negative sign is written and pronounced. Also note that zero is neither positive nor negative.

1-2 The Number Line:



A number line is used to illustrate the order of integers.

As we move to the right, the value of the integer increases, and as we move to the left, the value decreases.

This means that any positive integer is greater than any negative integer, and zero is greater than any negative number but less than any positive number.

For the numbers $\mathbf{5}$ and $-\mathbf{5}$, each is the additive inverse (or opposite) of the other.

In general, for any integer a, its opposite −a is its additive inverse, while zero is the additive inverse of itself.

On the number line, an integer and its additive inverse are represented by two points that are the same distance from zero but on opposite sides of it.

1-3 The Absolute Value of an Integer:

The absolute value of an integer a is denoted by $|a|$.

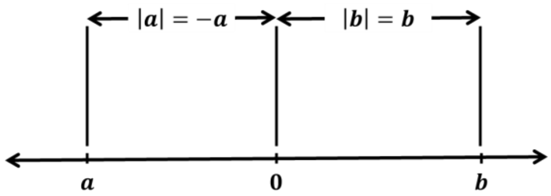
For example:

$$|5| = 5 \quad , \quad |-5| = 5 \quad , \quad |0| = 0$$

Geometrically, each integer is represented by a point on the number line.

The expression $|a|$ represents the distance between the point representing the integer a and the (zero) on the number line.

In general, $|a - b|$ represents the distance between the points representing the two integers a and b.



When finding the absolute value of an algebraic expression, if the result is negative, it is made non-negative by removing the negative sign (−).

1-4 Fundamental Rules of Addition, Subtraction, Multiplication, and Division:

Commutative Property:

- $a + b = b + a$
- $ab = ba$

Associative Property:

- $(a + b) + c = a + (b + c)$
- $(ab)c = a(bc)$

Distributive Property:

- $a(b + c) = (b + c)a = ab + ac$
- $a(b - c) = (b - c)a = ab - ac$

Closure Property:

The closure property holds for **addition**, **subtraction**, and **multiplication**, but not for **division**.

For any two integers $\boldsymbol{a}$ and $\boldsymbol{b}$, the results of $\boldsymbol{a}+b$, $\boldsymbol{a}-b$, and $\boldsymbol{a}\times b$ are always integers.

However, $\boldsymbol{a}\div\boldsymbol{b}$ is **not necessarily** an integer.

Additive Identity Property:

For any integer $\boldsymbol{a}\in\mathbb{Z}$, the number $\mathbf{0}$ is the additive identity, because:

- $\boldsymbol{a}+\mathbf{0}=\mathbf{0}+\boldsymbol{a}=\boldsymbol{a}$

Additive Inverse Property:

For every integer $\boldsymbol{a}\in\mathbb{Z}$, there exists an integer $-\boldsymbol{a}\in\mathbb{Z}$ such that:

- $\boldsymbol{a}+(-\boldsymbol{a})=(-\boldsymbol{a})+\boldsymbol{a}=\mathbf{0}$

1-5 Exponents:

An exponent is used to represent repeated multiplication.

For example:

$$\mathbf{2}\times\mathbf{2}\times\mathbf{2}=\mathbf{2}^{\mathbf{3}}$$

In general:

$$a^n=\overbrace{a\times a\times a\times\ldots\times a}^{n\ times}$$

Properties of Exponents (Powers):

If $x,y>0$ and $m,n\in\mathbb{Z}$, then:

(1) $x^m\cdot x^n=x^{m+n}$ (2) $\dfrac{x^m}{x^n}=x^{m-n}$ (3) $(x^m)^n=x^{mn}$

(4) $(xy)^n=x^ny^n$ (5) $\left(\dfrac{x}{y}\right)^n=\dfrac{x^n}{y^n}$ (6) $x^0=1$

(7) $x^{-1}=\dfrac{1}{x}$

1-6 Rule for Removing Parentheses:

For any two integers $\boldsymbol{x}$ and $\boldsymbol{y}$:

$$(1)\ \boldsymbol{x + (y) = x + y} \quad , \quad \boldsymbol{x + (-y) = x - y}$$

$$(2)\ \boldsymbol{x - (y) = x - y} \quad , \quad \boldsymbol{x - (-y) = x + y}$$

$$(3)\ \boldsymbol{x(-y) = (-x)y = -xy} \quad , \quad \boldsymbol{(-x)(-y) = xy}$$

$$(4)\ \begin{cases} (-1)^n = -1 & if\ \boldsymbol{n}\ is\ an\ \boldsymbol{odd}\ integer. \\ (-1)^n = 1 & if\ \boldsymbol{n}\ is\ an\ \boldsymbol{even}\ integer. \end{cases}$$

Examples:

(1) Find the value of each expression:

$\boldsymbol{a})\ (-2) + 12 =$ $\quad$ $\boldsymbol{b})\ (-3) + (-6) + 5 =$ $\quad$ $\boldsymbol{c})\ -3 - 11 - 31 =$

$\boldsymbol{d})\ (-5) \times (-4) =$ $\quad$ $\boldsymbol{e})\ (-4) \times 8 =$ $\quad$ $\boldsymbol{f})\ (-1) \times 2 \times 2 =$

Solution:

$\boldsymbol{a})\ (-2) + 12 = 10$

$$\boldsymbol{b})\ (-3) + (-6) + 5 = [(-3) + (-6)] + 5$$
$$= (-9) + 5 = -4$$

$\boldsymbol{c})\ -3 - 11 - 31 = -(3 + 11 + 31) = -45$

$\boldsymbol{d})\ (-5) \times (-4) = 20$

$\boldsymbol{e})\ (-4) \times 8 = -32$

$\boldsymbol{f})\ (-1) \times 2 \times 2 = -4$

(2) Find the value of each expression:

$\boldsymbol{a})\ |-2| + (-2) =$

$\boldsymbol{b})\ (-3)^2 - 3 =$

$\boldsymbol{c})\ (a^2)^3 \times a^3 =$

$\boldsymbol{d})\ \dfrac{x^3 \times x}{x^2} =$

Solution:

$\boldsymbol{a})\ |-2| + (-2) = 2 - 2 = 0$

$\boldsymbol{b})\ (-3)^2 - 3 = 9 - 3 = 6$

$\boldsymbol{c})\ (a^2)^3 \times a^3 = a^6 \times a^3 = a^9$

$\boldsymbol{d})\ \dfrac{x^3 \times x}{x^2} = \dfrac{x^4}{x^2} = x^2$

(3) Simplify:

$$3a + \{-4b - [4a - 7b - (-4a - b)] + 5a\}$$

Solution:

$$3a + \{-4b - [4a - 7b - (-4a - b)] + 5a\} = 3a + \{-4b - [4a - 7b + 4a + b] + 5a\}$$

$$= 3a + \{-4b - [8a - 6b] + 5a\}$$

$$= 3a + \{-4b - 8a + 6b + 5a\}$$

$$= 3a + \{2b - 3a\}$$

$$= 2b + (3a - 3a)$$

$$= 2b$$

Exercises:

(1) Find the value of each expression:

$\boldsymbol{a})\ (-4) + 9 =$

$\boldsymbol{b})\ -42 \div 7 =$

$\boldsymbol{c})\ (2)^5 =$

$\boldsymbol{d})\ (-4)^3 =$

$\boldsymbol{e})\ (-5)^2 =$

$\boldsymbol{f})\ |0| =$

$\boldsymbol{g})\ (-4) \times (-8) =$

$\boldsymbol{h})\ -8 - (-5) =$

$\boldsymbol{i})\ -1 - 4 + 7 =$

$\boldsymbol{j})\ 2 \times 4 + 6 \times 5 =$

$\boldsymbol{k})\ |-6| =$

$\boldsymbol{m})\ |-3| - |-7| =$

(2) Find the value of each expression:

$\boldsymbol{a})\ a^2 \times a^5 =$

$\boldsymbol{b})\ x^7 \div x^3 =$

$\boldsymbol{c})\ (a^3)^4 =$

$\boldsymbol{d})\ (x^2)^3 \times (x^4)^5 =$

$\boldsymbol{e})\ \dfrac{a^3 \times a^7}{a^2 \times a^6} =$

$\boldsymbol{f})\ \dfrac{a^4 \times a^5}{a^3 \times a^6} =$

(3) Compute:

$\boldsymbol{a})\ (-7) + (-12) - (-14) - (-15) - (-18) - (-38)$

$\boldsymbol{b})\ (-5)^2 + |-6| - (-1)^{1447}$

Challenge Problems:

(1) Compute:

$$-1-(-1)^1-(-1)^2-(-1)^3-\cdots-(-1)^{99}-(-1)^{100}$$

(2) Find the value of:

$$1234\times 9999=\cdots$$

(3) Simplify:

$$5\{(2a-3)-[7(4a-1)-20]\}-(3-8a)$$

(4) A satellite completes one full orbit around the Earth every 7 hours.

How many times will the satellite orbit the Earth in one week?

(5) Six fair dice were rolled. The sum of the numbers shown on all six faces is 32.

What is the smallest possible number that could appear on one of the dice?

(6) Compute:

$$1-2+3-4+\ldots-100+101$$

2-Rational Numbers $\mathbb{Q}$

A **rational number** is any number that can be written in the form

$$\frac{a}{b}$$

where a and b are integers, and $b \neq 0$.

The number a is called the numerator, and b is called the denominator.

From this definition, we can conclude that every integer is a rational number.

For example:

$$0 = \frac{0}{1}, -4 = \frac{-4}{1}, 5 = \frac{5}{1}$$

Also, the mixed number

$$5\frac{1}{2} = \frac{11}{2}$$

is a rational number because it can be written as a fraction.

And so on...

Terminating decimals are rational numbers. For example:

$$0 \cdot 3 = \frac{3}{10}, 1.7 = \frac{17}{10}, 2.03 = \frac{203}{100}$$

Non-terminating decimals are of two types:

Repeating decimals:

$$0.333 \ldots = 0.\overline{3}, \quad 0.521521521 \ldots = 0.\overline{521}$$

These are rational numbers because their digits repeat in a predictable pattern.

Non-repeating decimals:

$$0.345674215677589\ldots$$

These are irrational numbers, because their digits continue infinitely without any repeating pattern.

Note:

The approximate ratio known as pi (π) is an irrational number.

It is approximately equal to 3.14, or sometimes represented as

$$\frac{22}{7},$$

which is only an approximation.

2-1 Properties of Addition and Subtraction of Rational Numbers

The properties of addition for rational numbers are the same as those for integers.

But wait—when adding or subtracting two rational numbers, why do we always hear that we must "find a common denominator"? Let's explore this question.

Actually, the answer is not as simple as it seems. Let's take a closer look.

We all know that the sum of three camels and five camels is eight camels. In general:

$$3x + 5x = 8x$$

where the symbol x represents the **same quantity**.

Now, suppose we divide a pizza into seven equal and identical parts. Each part is called **one-seventh** and is written as:

$$\frac{1}{7}$$

However, when we use symbols, things may appear different:

$$2\left(\frac{1}{7}\right) + 3\left(\frac{1}{7}\right) = 5\left(\frac{1}{7}\right)$$

Since $2\left(\frac{1}{7}\right)$ can be written as $\frac{2}{7}$, we can express the equation in a simpler form:

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

Here, we added numerators because the **denominators are the same**.

(Notice that many students mistakenly try to add denominators as well — but denominators must remain the same!)

By similar reasoning, you can conclude that subtracting two rational numbers also requires the **same denominator**.

But what if the denominators are **different**?

How do we add or subtract rational numbers then?

We'll soon find the answer after studying the next property of rational numbers.

2-2 The Property of Equivalent Rational Numbers

If $\frac{a}{b}$ is a rational number and k is a nonzero integer, then:

$$\frac{a}{b} = \frac{a \cdot k}{b \cdot k} = \frac{a \div k}{b \div k}$$

This means that a rational number can be written in many equivalent forms.

For example:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10} = \cdots$$
$$\frac{18}{24} = \frac{9}{12} = \frac{3}{4}$$

A rational number whose numerator and denominator have no common factor (other than 1) is said to be in **simplest form**.

The simplest form of a rational number is the most convenient to use in calculations.

Therefore, it is always recommended to express any rational number in its simplest form.

Notice: Now You Can Add Two Rational Numbers with Different Denominators!

For example, when adding

$$\frac{2}{3}+\frac{1}{5}$$

we first look for a **common multiple** of the denominators 3 and 5 (preferably the least common multiple).

It is easy to see that 15 is a multiple of both 3 and 5.

Now, we rewrite the fractions as **equivalent fractions** with a common denominator:

$$\frac{2\times 5}{3\times 5}+\frac{1\times 3}{5\times 3}$$

That is,

$$\frac{2}{3}+\frac{1}{5}=\frac{10}{15}+\frac{3}{15}$$

and this equals

$$\frac{13}{15}$$

Subtraction is handled in the **same way** as addition.

2-3 Definition of Multiplication and Division in Rational Numbers

How do we multiply two rational numbers, and what does that mean?

For example, when we ask, *"What is half of 100?"* you immediately answer **50** — as if you divided **100** by **2**.

Now, if we ask, *"What is three halves of 100?"* it's not difficult to find the answer: you first divide by **2**, then multiply the result by **3**.

Let's ask a more complex question:

What is three halves of the number $\frac{1}{7}$?

We need to divide the number by **2**, and then multiply the result by **3**.

But what is the result of dividing $\frac{1}{7}$ by 2?

$$\frac{1}{7} = \frac{2}{14} = 2(\frac{1}{14})$$

So, if the result of dividing $\frac{1}{7}$ by **2** is $\frac{1}{14}$ (that is, only the denominator is multiplied by **2**), then multiplying by **3** means making three copies of that result:

$$3(\frac{1}{14}) = \frac{3}{14}$$

Let's express what we found symbolically:

$$100 \times \frac{3}{2} = 50 \times 3 = 150$$

Similarly,

$$\frac{1}{7} \times \frac{3}{2} = \frac{1 \times 3}{7 \times 2} = \frac{3}{14}$$

In general, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Finally, since division is the inverse operation of multiplication, we can justify the following definition:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

2-4 Properties of Multiplication of Rational Numbers

1. **Closure Property:**

   The product of any two rational numbers is also a rational number.

2. **Commutative Property:**

Changing the order of the factors does not change the product.

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$$

3. **Associative Property**:

The way the factors are grouped does not affect the product.

4. **Multiplicative Identity:**

The number 1 is the multiplicative identity because multiplying any rational number by 1 does not change its value.

5. **Multiplicative Inverse:**

For every non-zero rational number $\frac{a}{b}$, there exists a number $\frac{b}{a}$ called its **multiplicative inverse**, such that:

$$\frac{a}{b} \times \frac{b}{a} = \frac{b}{a} \times \frac{a}{b} = 1$$

6. **Distributive Property of Multiplication over Addition and Subtraction**

2-5 Writing a Repeating Decimal as a Rational Number

Example:

Write the repeating decimal $\mathbf{0.\overline{7}}$ as a rational number.

Solution:

Let

$$x = 0.\overline{7} = 0.7777\ldots$$

Multiply both sides by **10**:

$$10x = 7.7777\ldots = 7 + 0.7777\ldots$$

Now we can write:

$$10x = 7 + x$$

Subtract x from both sides:

$$9x = 7$$

Finally, divide both sides by 9 to find:

$$x = \frac{7}{9}$$

Practice:

Write the repeating decimal $0.\overline{17}$ as a rational number.

Hint: *Follow the same steps as in the example, but this time you will need to multiply by 100 instead of 10.*

A Quick Method for Writing a Repeating Decimal as a Rational Number

(The proof is not difficult.)

We can write a repeating decimal as a fraction directly, provided that the repeating part starts immediately after the decimal point.

To do this, place the repeating digits in the numerator, and write as many nines in the denominator as there are repeating digits.

For example:

$$0.\overline{7} = \frac{7}{9}, \qquad 0.\overline{73} = \frac{73}{99}, \qquad 0.\overline{732} = \frac{732}{999}, \qquad \text{and so on.}$$

This method can also be extended to write any other repeating decimal as a rational number. For example:

$$\begin{aligned} 0.1\overline{7} &= 0.1 + 0.0\overline{7} \\ &= \frac{1}{10} + \frac{1}{10} \times \frac{7}{9} \\ &= \frac{1}{10} + \frac{7}{90} \\ &= \frac{9}{90} + \frac{7}{90} \\ &= \frac{16}{90} \\ &= \frac{8}{45} \end{aligned}$$

Exercises:

(1) Find the value of each of the following:

$(a)\ \frac{2}{5}+\frac{1}{6}=$

$(b)\ \frac{1}{2}-\frac{1}{3}=$

$(c)\ \frac{-3}{8}\times\frac{4}{9}=$

$(d)\ \frac{-3}{5}-\left(-\frac{1}{2}\right)$

$(e)\ 1\div 3\div 4\div 5=$

$(f)\ \frac{1}{9}-\frac{1}{10}=$

$(g)\ (-3)\div 4\times 6\div(-5)=$

$(h)\ \frac{-6}{35}\div\frac{2}{7}=$

$(i)\ \left(\frac{-1}{2}\right)^{5}=$

(2) Write each of the following repeating decimals as a rational number:

$a)\ 0.\overline{2}$ $\quad b)\ 0.\overline{37}$ $\quad c)\ 1.8\overline{23}$

(3) Majed claims that dividing by an integer is equivalent to multiplying by its multiplicative inverse.

Can you prove or disprove this claim?

(4) Saleh claims that between any two rational numbers, there is always another rational number.

He also states that this property is called the **density of rational numbers**.

Can you give an example that supports his claim, or provide a counterexample if you think otherwise?

(5) Arrange the following numbers in ascending order:

$$-0.3\ ,-0.23\ ,-\frac{1}{3}$$

Challenge Problems:

(1) Find a fraction that is greater than $\frac{1}{6}$ and less than $\frac{1}{3}$, with a denominator of **15**.
(2) Saad divided a number N by **8** and got **0.25**, while Khaled multiplied the same number N by **8**. What result will Khaled obtain?
(3) Haitham went jogging outside his house. He has already completed $\frac{3}{5}$ of the second half of his route. What fraction of the entire route has he completed?
(4) Compute: $(1-\frac{1}{2})+(\frac{1}{2}-\frac{1}{3})+(\frac{1}{3}-\frac{1}{4})+\ldots+(\frac{1}{9}-\frac{1}{10})$
(5) Compute: $\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\ldots\cdot\frac{79}{80}$
(6) Adel has a card containing **14** digits. If the sum of any **3** consecutive digits (starting from the left) is **20**, he wrote down the digits on his card as shown below. What number should be written in the square labeled **A**? | A | | 7 | | | | | | | | | 7 | | 4 |
(7) Today is the birthday of **Laila** and her two daughters, **Shahd** and **Amal**. Laila is **32 years old**, Shahd is **4 years old**, and Amal is **1 year old**. How old will **Laila** be when **her age** becomes equal to **the sum of Shahd's and Amal's ages**?

(8) My uncle lives far away in an isolated place, and his letters to us always contain riddles.

In one of his letters, he told us that there are **three local teams** in his area: the **Ants (A)**, the **Bees (B)**, and the **Cats (C)**.

Each year, they play a **local league**, where every team plays against each of the others **at most once**.

The league table for this year began as follows:

Team	Played	Won	Drawn	Lost	Goals For	Goals Against
A	1	0	0	1	4	2
B	2	1	1	0	2	2
C	2	1	0	1	3	1

When we pointed out that these numbers were impossible, my uncle admitted that **every number in the table was incorrect**,

but that **each number should either have 1 added to it or 1 subtracted from it**.

Find the **correct table**, and explain clearly **how you deduced your results**.

3-Linear Equations in One Variable

To solve a linear equation in one variable, follow the steps below in an organized way:

1. Eliminate the denominators (if any)

If the equation contains fractions, multiply each term by the *Least Common Multiple (L.C.M)* of the denominators to eliminate them.

2. Remove the parentheses

Use the *distributive property* to simplify the parentheses.

Example:

$$2(x+3) = 2x+6$$

3. Transpose the terms

Move all terms containing the variable to one side of the equation and all constants to the other side according to the following rule:

When moving a term from one side of the equation to the other, its **sign changes**, while the terms that remain on the same side keep their signs.

4. Combine like terms

After transposing, simplify the equation to get it in the form:

$$ax = b$$

where a and b are constants.

5. Divide by the coefficient of the variable

Divide both sides by a (the coefficient of the variable) to find the solution:

$$x = \frac{b}{a}$$

Special Cases

- If $\boldsymbol{a} = \mathbf{0}$ and $\boldsymbol{b} \neq \mathbf{0}$, **there is no solution**.
- If $\boldsymbol{a} = \mathbf{0}$ and $\boldsymbol{b} = \mathbf{0}$, **any real number** is a solution to the equation.

6. Verify the solution

Substitute the obtained value of the variable into the *original equation*.

If both sides are equal, the solution is correct.

If not, an error occurred in the steps.

Note:

Sometimes we do not obtain an exact numerical solution; in such cases, we find the **approximate** value of the solution.

Example:

> **Solve the** equation:
>
> $a)\ x + 3 = 4$ $\qquad$ $b)\ x - 2 = 9$ $\qquad$ $c)\ 7x = 49$

Solution:

$a)\ x = 1$ $\qquad$ $b)\ x = 11$ $\qquad$ $c)\ x = 7$

Exercises:

(1) Solve the following equations:

$\boldsymbol{a}$) $x+3=4$ $\boldsymbol{b}$) $x-2=9$ $\boldsymbol{c}$) $7x=49$

$\boldsymbol{d}$) $\frac{x}{3}=6$ $\boldsymbol{e}$) $2x-1=19$ $\boldsymbol{f}$) $4x-4=x+11$

$\boldsymbol{g}$) $9(x-1)=7(x+1)$

(2) Solve the equation:

$$\frac{\mathbf{1}}{\mathbf{7}}(\mathbf{5}\boldsymbol{x}+\mathbf{2})=\mathbf{1}$$

(3) Solve the equation:

$$\frac{1}{2}\left[\frac{1}{7}(5x-1)\right]+5=6$$

(4) Word Problems Solved Using Linear Equations:

a) If 7% of a certain number equals 56, find the number?

b) The sum of four consecutive natural numbers is 50. Find the largest number?

c) Two natural numbers are in the ratio 2: 3, and their difference is 14. What is the smaller number?

d) Ahmad bought a computer at a 35% discount off its original price and paid 1300 riyals. What was the original price of the computer?

e) Mohammed bought a car and later sold it for 46,000 riyals, making a profit of 15%. What was the purchase price of the car?

Challenge Problems:

(1) A train was observed moving at a constant speed. It took 1 minute to completely pass through a tunnel that is 120 meters long. If the train, moving at the same constant speed, takes 20 seconds to completely pass a signal post, find the length of the train.
(2) A jar contains red and white balls in the ratio 1 : 4. When Saad replaces 2 white balls with 7 red balls, the ratio of red balls to white balls becomes 2 : 3. What is the ratio of the total number of balls now to the total number of balls originally?
(3) Laila gave birth to her first child on her 20th birthday. Her second child was born exactly two years later, and her third child was born exactly two years after that. At what age will the sum of her children's ages equal her own age?
(4) A basket contains apples and oranges. The ratio of apples to oranges in the basket is 3 : 8. After one apple is taken out of the basket, the ratio of apples to oranges becomes 1:3. How many oranges are there in the basket?
(5) The arithmetic mean of six numbers is 4. When a seventh number is added, the new mean becomes 5. Find the seventh number.
(6) A coach was buying 17 soccer balls for his club. The price of one ball was 481 riyals. The seller told him, “You need to pay 10,177 riyals for all the balls.” How could the coach suspect that he was being overcharged without performing any complicated calculations?

(7) The sum of two numbers was multiplied by their product.

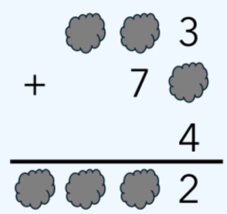
Could the result be 20042401?

If so, give an example. If not, explain why.

(8) Suad wrote a vertical addition problem in which all the digits were different.

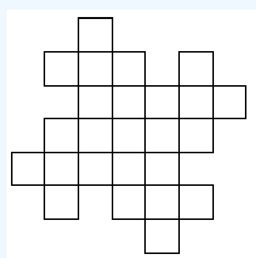
After she proudly showed it to her brother, he accidentally spilled ink on her notebook, covering some of the digits.

Can you help Suad rewrite the correct addition problem?



(9) Color five squares in the figure below so that the remaining part can be divided into five congruent pieces.

(Providing one example is enough to answer the question.)

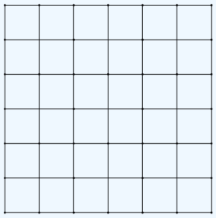


(10) A farmer bought three cows. Now each day he gets:

- **From the first cow:** 2 liters of milk.
- **From the second cow:** Half the amount of milk obtained from the third cow plus the same amount obtained from the first cow.
- **From the third cow:** Half of the total amount of milk obtained from all three cows.

How many liters of milk does the farmer get in total each day?

(11) Color six squares in the grid below black so that it becomes impossible to find any white strip of size 1 × 6 or any white square of size 3 × 3 after coloring.



(12) In the figure below, the addition of three numbers is shown.

Each letter represents one of the digits from 0 to 9 and stands for the same digit each time it appears.

Different letters represent different digits.

The leftmost digit of the result is not 0.

$$\begin{array}{r} J\ M\ O \\ J\ M\ O \\ +\ J\ M\ O \\ \hline I\ M\ O \end{array}$$

Find all possible values of the sum.

Second Unit: Geometry

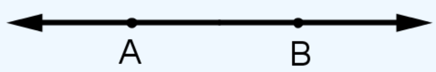
1-Points, Lines, and Angles

Geometry starts with three simple objects: points, lines, and angles. They are so simple that you already see them every day, but so powerful that they can build entire worlds. Let us start slowly and make friends with them.

Definitions:

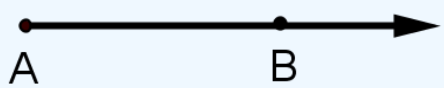
Definition 1.1

A *line* is a perfectly straight path that extends without end in both directions. It has no thickness and no boundaries.



Definition 1.2

A *ray* begins at one point (called its *endpoint*) and extends forever in one direction. It is like a flashlight beam: it starts at the bulb and travels infinitely forward.

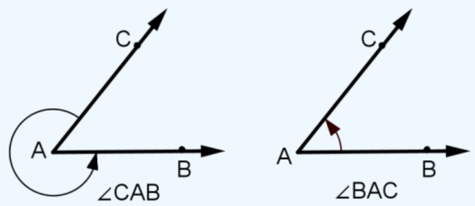


Definition 1.3

A *segment* is the part of a line between two points. It has a beginning and an end, just like a stick you can hold.

Definition 1.4

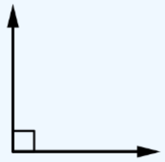
An *angle* is formed when two rays meet at a common endpoint (the *vertex*). Angles can be measured in two directions: clockwise (CW) and counterclockwise (CCW). To avoid confusion, mathematicians usually use CCW as the "positive" direction.



Types of angles:

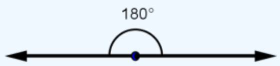
Acute: less than 90° (sharp like a needle).

Right: exactly 90° (corner of a book).



Obtuse: between 90° and 180° (wide and open).

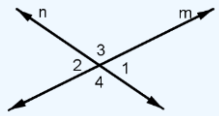
Straight: exactly 180° (a line).



Complementary: two angles adding up to 90°.

Supplementary: two angles adding up to 180°.

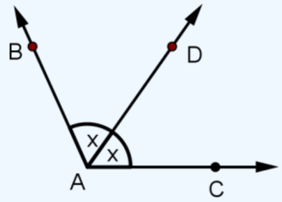
Adjacent: angles sharing a vertex and a side.



Vertical: opposite angles formed when two lines cross; they are always equal.

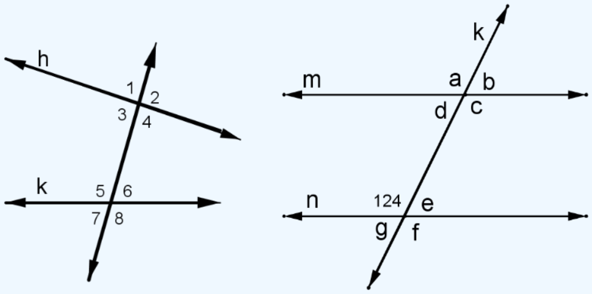
Definition 1.5

An *angle bisector* is a ray that divides an angle into two equal smaller angles. It is the "fair splitter" of angles.



Definition 1.6

Two lines are *parallel* if they never meet, no matter how far they are extended.

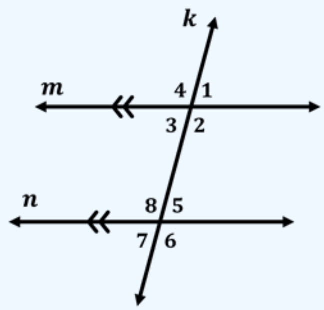


Definition 1.7

A *secant line* cuts across another figure (such as a circle), touching it at two points.

Definition 1.8

When a line (called a *transversal*) crosses parallel lines, many special angles appear:



Corresponding angles: same side, same position. (like $\angle 1\ and\ \angle 5$ or $\angle 2\ and\ \angle 6$)

Alternate interior angles: opposite sides of the transversal, between the parallels. (like $\angle 3\ and\ \angle 5$ or $\angle 2\ and\ \angle 8$)

Alternate exterior angles: opposite sides of the transversal, outside the parallels. (like $\angle 1\ and\ \angle 7$ or $\angle 4\ and\ \angle 6$)

Consecutive interior angles: same side of transversal, between the parallels. (like $\angle 2\ and\ \angle 5$ or $\angle 3\ and\ \angle 8$)

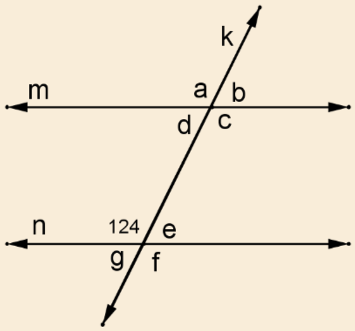
Theorem 1: If two lines are parallel, then corresponding angles are equal.

Theorem 2: If corresponding angles are equal, then the lines are parallel.

Example

On the provided figure, lines m, n are parallel. And k is a traversal line. We have a measure of one angle as shown. Find the measure of the angles:

$$\angle a, \angle b, \angle c, \angle d, \angle e, \angle f, \angle g$$



Solution:

Since m, n are parallel, we have the following measures:

$\angle a = 124°$ corresponding angle

$\angle b = 56°$ supplementary to $\angle a$

$\angle c = 124°$ vertical angle to $\angle a$

$\angle d = 56°$ supplementary to $\angle a$

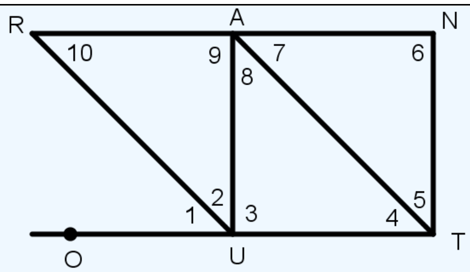
$\angle e = 12$ corresponding to angle $\angle b$

$\angle f = 124°$ corresponding to angle $\angle c$

$\angle g = 56°$ corresponding to angle $\angle d$

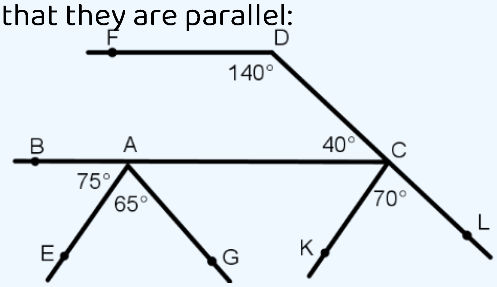
Exercises

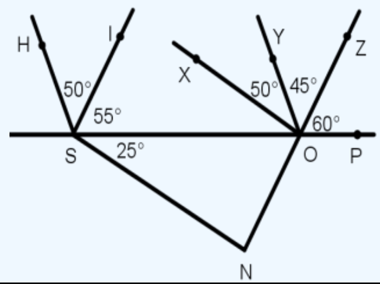
(1) On the figure below, find all pairs of parallel lines in each of the following cases:



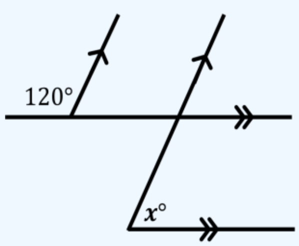
a) $\angle 1 \simeq \angle 4$
b) $\angle 2 \simeq \angle 10$
c) $\angle 5 \simeq \angle 7$
d) $\angle 5 \simeq \angle 8$
e) $\angle 6 \simeq \angle 9 = 90°$
f) $\angle 6 \simeq \angle 3 = 90°$
g) $\angle 7 \simeq \angle 10 \simeq \angle 1$

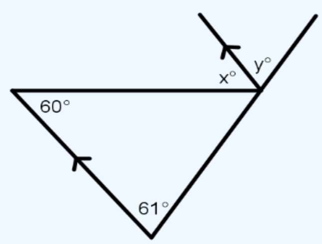
(2) In the following figures, find the parallel lines and determine which angles were used to show that they are parallel:



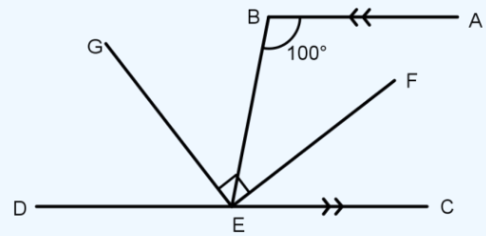


(3) In the following figures, find the measure of the unknown angles:

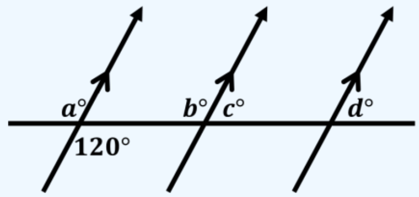




(4) In the following figure, we have $AB \parallel DC$. And EF bisects $\angle BEC$. Find the measure of $\angle BEG, \angle DEG$.

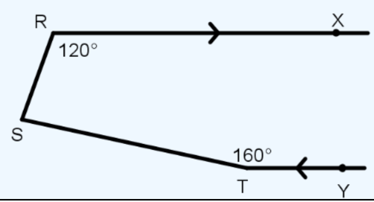


($\mathbf{5}$) In the following figure, Find the measure of $\angle d$.

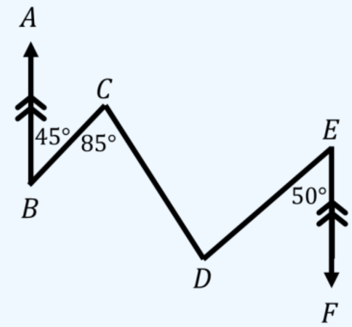


(6) In the following figure, Find the measure of $\angle RST$.

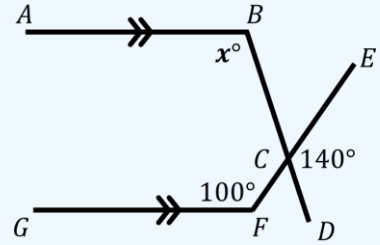
(Hint, draw a line passing through S and parallel to lines $\overline{RX}, \overline{TY}$.)



($\mathbf{7}$) In the following figure, we have $\overrightarrow{BA} \parallel \overrightarrow{EF}$, find the measure of $\angle CDE$.

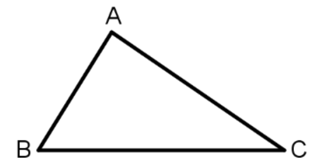


($\mathbf{8}$) In the following figures, find the measure of $\angle x$:

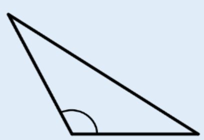
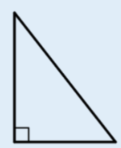
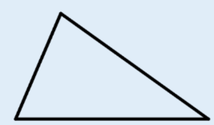
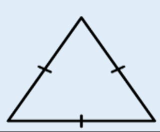
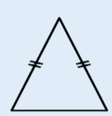
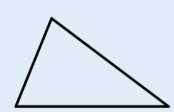


2-Triangles

Triangles are one of the core shapes in geometry and is a special case of polygons.



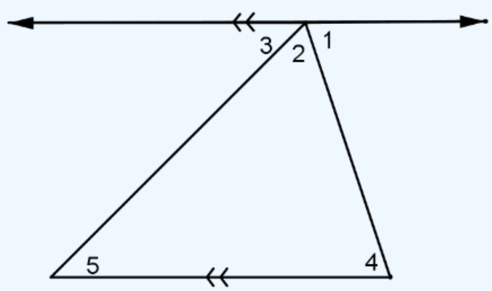
The symbol △ is usually used to denote a triangle. The triangle is built using three different segments. The three vertices connecting the segments are usually called A, B, C and the three segments are called sides and usually denoted by $\overline{AB}$,$\overline{BC}$,$\overline{CA}$. Finally, the three angles of the triangle are denoted by $\angle A$, $\angle B$, $\angle C$.

Obtuse Triangle	Right Triangle	Acute Triangle
		
one angle greater than 90°	one angle is exactly 90°	all angles less than 90°
	The side opposite to the 90° angle is called the hypotenuse while both of the other sides adjacent to the 90° angle are called bases/legs	
Equilateral Triangle	Isosceles Triangle	Scalene Triangle
		
all three sides equal	two equal sides	all three sides different
In this case, all angles are equal to 60°	The two angles opposite to the equal sides are also equal. The two equal sides are called legs while the third side is called the base.	

Theorem:

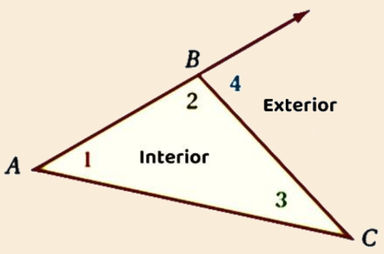
The three interior angles of any triangle add up to $180°$.

Proof. To prove that, draw a line passing through a vertex parallel to the side opposite to that vertex. Since angles $\angle 1, \angle 2, \angle 3$ lie on a straight line, they sum up to $180°$. Finally, notice that $\angle 1 \simeq \angle 4$ and $\angle 3 \simeq \angle 5$ by alternate interior angles. Therefore $\angle 2 + \angle 4 + \angle 5 = 180°$.



From this theorem, many facts follow naturally:

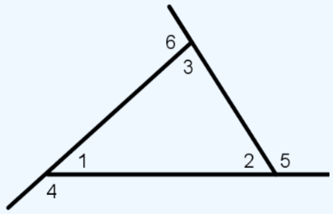
- Extending one side forms an exterior angle, which equals the sum of the two non-adjacent interior angles.
- If two angles are equal, the opposite sides are equal, and vice versa.
- A triangle can have at most one right or one obtuse angle.
- In a right triangle, the other two angles are acute and together add up to $90°$.



Exercises

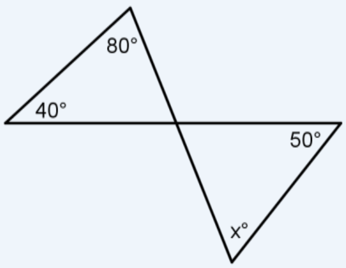
(1) Using the adjacent figure, calculate the following angles:

1. If $\angle 1 = 40°, \angle 2 = 60°$, then find $\angle 6$.
2. If $\angle 1 = 45°, \angle 3 = 70°$, then find $\angle 5$.
3. If $\angle 2 = 50°, \angle 3 = 65°$, then find $\angle 4$.
4. If $\angle 4 = 135°, \angle 2 = 60°$, then find $\angle 3$.
5. If $\angle 5 = 120°, \angle 1 = 40°$, then find $\angle 3$.
6. Find $\angle 4 + \angle 5 + \angle 6$

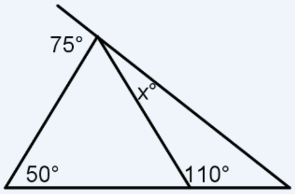


In $(\mathbf{2}-\mathbf{5})$ find the value of $\boldsymbol{x}$ from the given figure:

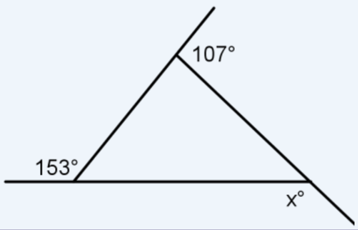
(2)



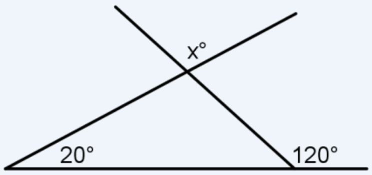
(3)



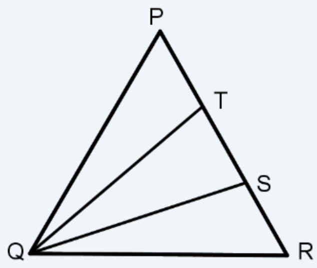
(4)



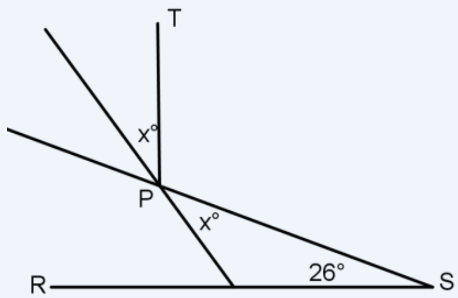
(5)



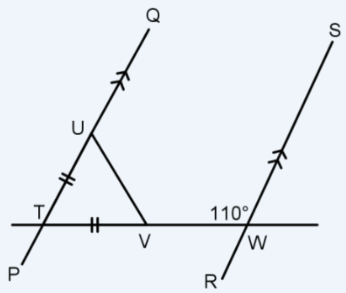
(6) In the adjacent figure: $\triangle QPR$ is an equilateral triangle. QT, QS divides $\angle PQR$ into three equal parts. Find the measure of $\angle QTP$.



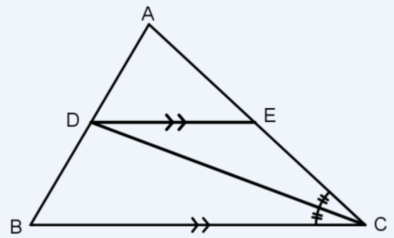
(7) In the adjacent figure: A light ray exits from point S, reflects at point P, and exits in the direction of point T such that RS is perpendicular to PT. Find the value of x.



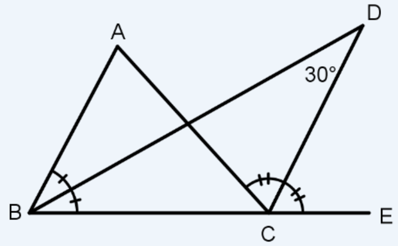
(8) In the adjacent figure, lines $RS \parallel PQ$, and $TU = TV$, and $\angle SWV = 110°$. Find $\angle QUV$.



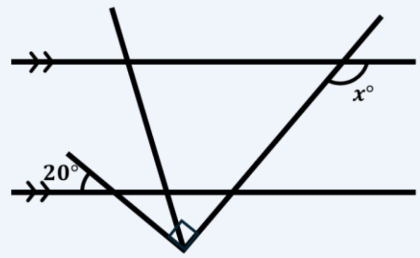
(9) in the adjacent figure: CD bisects $\angle ACB$, and $\angle ACB = 40°$, and $\angle B = 70°$, and $DE \parallel BC$. Find the measures of both of $\angle EDC$, $\angle BDC$.



(10) In the adjacent figure: Find the value of x.

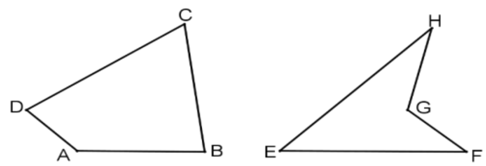


(11) in the adjacent figure: The bisector of $\angle ABC$ and the bisector of $\angle ACE$ intersect at point D. If $\angle BDC = 30°$, find the measure of $\angle A$.



3-Polygons

A *polygon* is a closed figure made by joining straight segments end to end. The corners are called *vertices*, and the sides are the segments themselves. A *diagonal* is a segment joining two non-adjacent vertices.



Definition 3.1

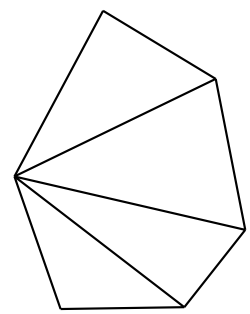
A polygon is *convex* if all its interior angles are less than $180°$. If one of the interior angles is greater than $180°$, it is *concave*.

Thus, a triangle is a 3-sided polygon, a quadrilateral has 4 sides, and so on.

Theorem 3.1

The sum of the interior angles of an n-sided polygon is $(n-2)\times 180°$.

Proof. Choose any of the polygon vertices, and draw all of its diagonals from that vertex.



This divides the polygon into $(n-2)$ triangles. Each triangle has an angle sum of $180°$. So, the total angle sum is:

$$(n-2)\times 180°$$

Definition 3.2

A polygon is *regular* if all its sides and all its angles are equal.

Theorem 3.2

Each interior angle of a regular n-sided polygon measures

$$\frac{(n-2)\times 180°}{n}.$$

Exercises:

(1) Find the sum of interior angles of a polygons with number of sides:

$a)$ 15

$b)$ 16

$c)$ 17

(2) Find the measure of the interior angle of a regular polygon of sides:

$a)$ 15

$b)$ 16

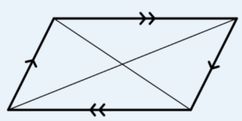
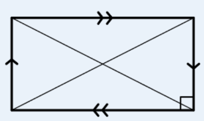
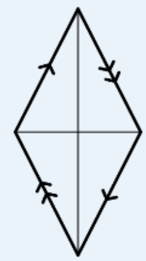
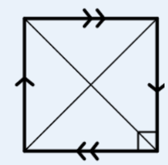
$c)$ 17

(3) Find the sum of exterior angles of a regular polygon.

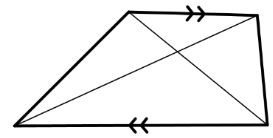
Solution: The sum is always 360^o no matter how many sides we have.

4-Quadrilaterals

Quadrilaterals are polygons with four sides. They come in many “personalities.”

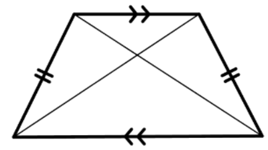
	Parallelogram	Rectangle	Rhombus	Square
Shape				
Definition	A quadrilateral with both pairs of opposite sides parallel.	A parallelogram with one right angle; all angles are right angles.	A parallelogram with all sides equal.	A rectangle with all sides equal.
Sides	Opposite sides equal.	Opposite sides equal.	All sides equal.	All sides equal.
Angles	Opposite angles equal.	All angles are right (90°).	Opposite angles equal.	All angles are right (90°).
Diagonals	Bisect each other.	Equal and bisect each other.	Bisect at right angles; vertices-connecting diagonal bisects angles.	Equal, bisect each other, perpendicular, angle-bisecting.
Area	Base × height	Length × width	Base × height	Side^2
Perimeter	Sum of all sides	2 × (length + width)	4 × side	4 × side

Definition 4.1



A trapezoid (or trapezium) is a quadrilateral with at least one pair of parallel sides.

Definition 4.2



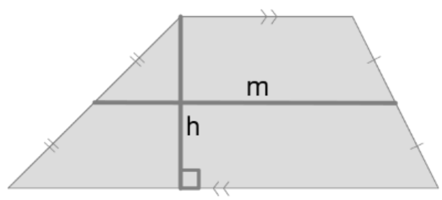
If the non-parallel sides (the *legs*) are equal, the trapezoid is called *isosceles*.

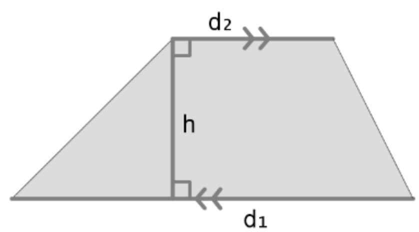
Theorem 4.1

The area of a trapezoid with bases b_1 and b_2, and height h, is

$$Area = \frac{(b_1 + b_2)}{2} \cdot h.$$

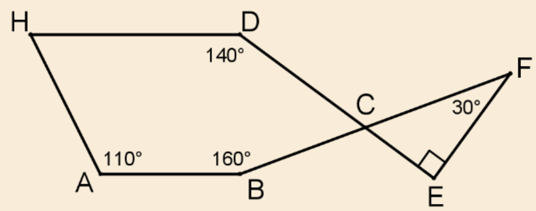
or using the mid-segment : $A = h \cdot m$





Example:

In the figure below: we have two lines DE, BF intersecting at point C, such that EF is perpendicular to DE, and $\angle ABC = 160°$, and $\angle HAB = 110°$. Prove that $AB \parallel HD$.



Solution:

In the right triangle $\triangle FEC$ at $\angle C$, since $\angle CFE = 30°$,

$$\angle FCE = 180° - (30° + 90°) = 60°$$

So by vertical angles, $\angle DCB = 60°$. And since the shape $ABCDH$ is a pentagon, then the sum of its interior angles is:

$$\angle A + \angle B + \angle C + \angle D + \angle H \ 180°(n - 2) \ \ 180°(5 - 2) \ \ 180° \cdot 3 \ \ 540°$$

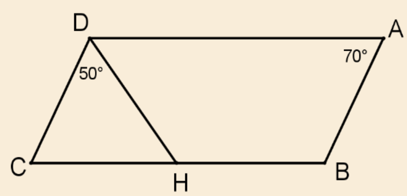
Therefore,

$$\angle H \ 540° - (110° + 160° + 60° + 140°) \ \ 540° - 470° = 70°$$

And since $\angle H + \angle A = 70° + 110° = 180°$ and both are internal angles lying on the same side of the transversal AH, then $AB \parallel HD$.

Example:

In the figure: $ABCD$ is a parallelogram where H lies on side BC, and $\angle A = 70^{\circ}$, $\angle CDH = 50^{\circ}$. Find the measure of $\angle BHD$.



Solution:

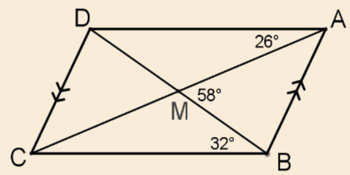
Since $ABCD$ is a parallelogram, then $\angle C = \angle A = 70^{\circ}$ (opposite angles in a parallelogram).

And since $\angle DHB$ lies outside $\triangle DHC$, then

$$\angle DHB \ \angle C + \angle CDH \ \ 50^{\circ} + 70^{\circ} \ \ 120^{\circ}$$

Example:

In the figure: $ABCD$ is a quadrilateral with diagonals intersecting at M, and $AB \parallel CD$, $\angle AMB = 58^\circ$, $\angle MBC = 32^\circ$. Prove that $ABCD$ is a parallelogram.



Solution:

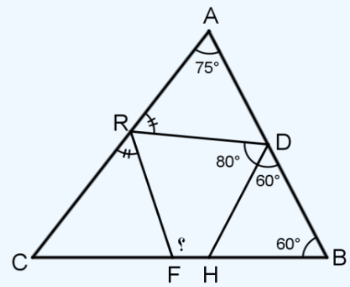
Since $\angle AMB$ lies outside triangle $\triangle CMB$, then $\angle AMB = \angle MBC + \angle MCB$. Therefore,

$$58^\circ = 32^\circ + \angle MCB \Longrightarrow \angle MCB = 26^\circ$$

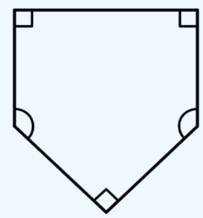
and $\angle CAD, \angle MCB$ are angles in an alternate position, so $AD \parallel BC$ and we already have from the given that $AB \parallel CD$ which implies $ABCD$ is a parallelogram.

Exercises

(1) In the figure: $\triangle ABC$, where $\angle BDH = \angle ABC = 60^{\circ}$ and $\angle HDR = 80^{\circ} = \angle BAC = 75^{\circ}$. Find the measure of $\angle HFR$.



(2) In American baseball, the base is shaped as a pentagon (as shown in the adjacent figure) consisting of three right angles and two identical angles. Find the measure of these two angles.

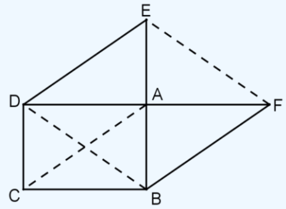


(3) The surface areas of a rhombus and an isosceles trapezoid are equal. If the side length of the rhombus is 10 and the length of the mid-base of the trapezoid is 15, find the ratio between their heights.

Note: The mid-base of a trapezoid is the segment connecting the midpoints of the non-parallel sides.

(4) A trapezoid with one of its parallel bases equal to 3 times the length of the other. If its height equals the length of its mid-base and its area is 100, find the lengths of the two parallel bases.

(5) In the figure below: $ABCD$ is a rectangle, and $ACBF$, $ACDE$ are parallelograms. Prove that $EF \parallel BD$.



5-Triangle Congruence

If we draw two triangles, let's say the side lengths of the first are 4,5,7, and then we draw the second triangle using the exact same side lengths on a transparent sheet.

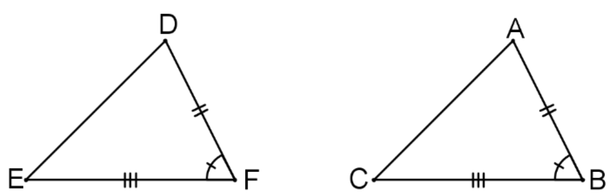
Placing the transparent sheet (which holds the second triangle) directly over the sheet containing the first triangle, we would observe that the two triangles perfectly coincide (superpose). We would also notice that all of the corresponding angles are equal in measure.

This phenomenon is what we call triangle congruence. The example mentioned here illustrates one specific case of congruence, and we will now proceed to discuss the cases of congruence in detail

Cases of Triangle Congruence

<u>First Case</u>:

Two triangles are congruent if two sides and the included angle of the first triangle are congruent (equal in measure) to the corresponding two sides and the included angle of the second triangle. We will refer to this case of congruence using the abbreviation $\boldsymbol{SAS}$. (side-angle-side)



In triangles $\triangle ABC, \triangle DFE$, if:

$$\begin{cases} AB = DF \\ BC = FE \\ \angle B \cong \angle F \end{cases}$$

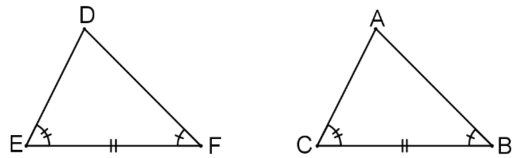
Then $\triangle ABC \cong \triangle DFE \ (SAS)$

And thus:

$$\begin{cases} AC = DE \\ \angle A \cong \angle D \\ \angle C \cong \angle E \end{cases}$$

Second Case:

Two triangles are congruent if two angles and the included side (the side connecting the vertices of the two angles) in one triangle are congruent (equal in measure and length) to the corresponding two angles and included side in the other triangle. We will call this SAS.



In triangles ABC, DFE, if:

$$\begin{cases} CB = EF \\ \angle C \cong \angle E \\ \angle B \cong \angle F \end{cases}$$

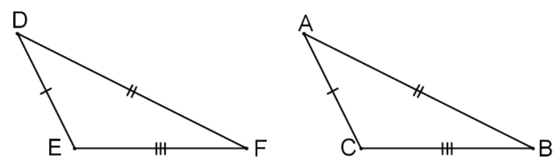
Then $\triangle ABC \cong \triangle DFE (ASA)$

Moreover, we get that:

$$\begin{cases} AB = DF \\ AC = DE \\ \angle A \cong \angle D \end{cases}$$

Case Three:

Two triangles are congruent if all three sides of the first triangle are congruent (equal in length) to the corresponding three sides of the second triangle. We will refer to this by SSS.



In triangles ABC, DFE, if:

$$\begin{cases} AB = DF \\ AC = DE \\ BC = EF \end{cases}$$

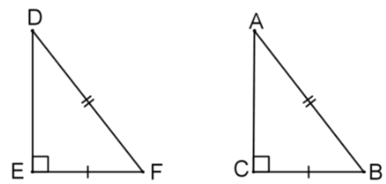
Then $\triangle ABC \cong \triangle DFE (SSS)$.

Moreover, we get that:

$$\begin{cases} \angle A \cong \angle D \\ \angle C \cong \angle E \\ \angle B \cong \angle F \end{cases}$$

Forth Case:

Two right-angled triangles are congruent if the hypotenuse and one leg (side forming the right angle) of the first triangle are congruent (equal in length) to the corresponding hypotenuse and one leg of the second triangle. We will refer to this by HS (hypotenuse-side).



In Triangles ABC, DFE, If:

$$\begin{cases} AB = DF \\ CB = EF \\ m(\angle C) = m(\angle E) = 90° \end{cases}$$

Then $\triangle ABC \cong \triangle DFE (HS)$,

Moreover, we get that:

$$\begin{cases} AC = DE \\ \angle A \cong \angle D \\ \angle B \cong \angle F \end{cases}$$

Remarks:

The two congruent sides in an isosceles triangle are called the legs. The third side, which is not congruent to the others, is called the base.

Isosceles Triangle Theorem:

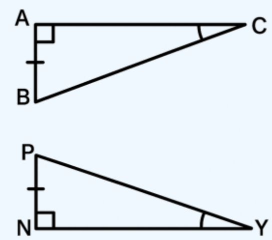
The angles of the base of an isosceles triangle are equal. And the opposite is true, if two angles of a triangle are equal, then the triangle is isosceles.

Conclusions:

- The angles of an equilateral triangle are all equal,
- And there measure is 60^o degrees.
- The angle bisector of the vertex of an isosceles triangle is perpendicular to the base and also bisects the base.
- The triangle with equal angles is also an equilateral triangle (has equal sides).

Exercises:

In the following figures, find (if it exists) the triangle that is congruent to ABC and state the postulate you used.

(1)


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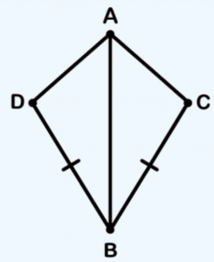

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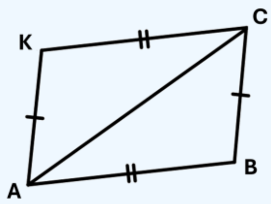

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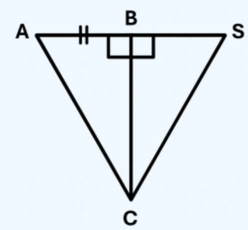

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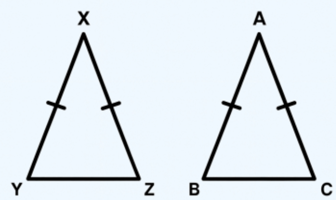

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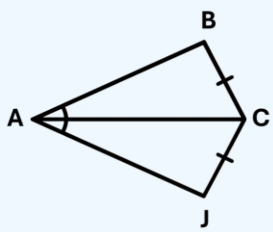

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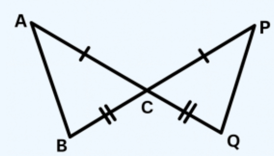

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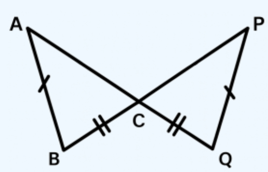

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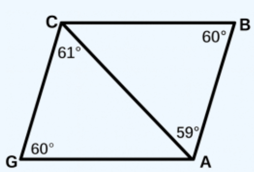

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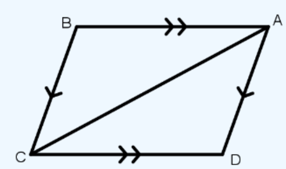

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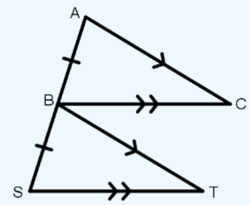

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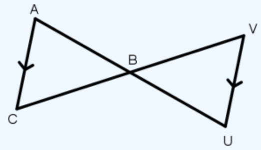

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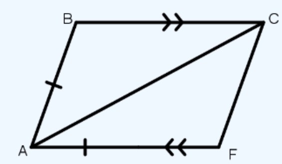

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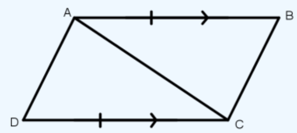

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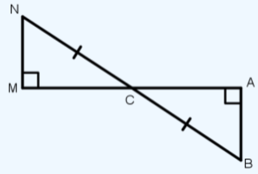
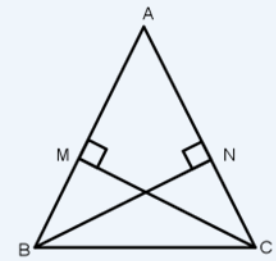


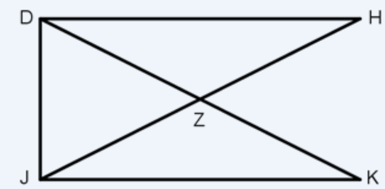
(16) In the adjacent figure: if $\overline{AB} \cong \overline{AC}$, and $\overline{BN} \perp \overline{AC}$, $\overline{CM} \perp \overline{AB}$, explain how you can prove that $\Delta ABN \cong \Delta ACM$



(17) In the adjacent figure:

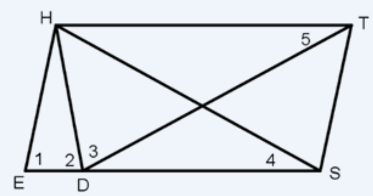
if $JH = DK, DH \perp DJ, JK \perp DJ$

prove that $\angle H = \angle K$

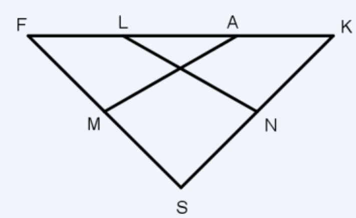


(18) In the adjacent figure: if $ES = DT, \angle 1 = \angle 2 = \angle 3$,

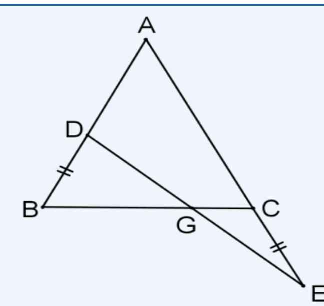
prove that $\angle 4 \cong \angle 5$



(19) In the adjacent figure: if $FL = AK, SF = SK$, and M is the midpoint of SF, N is the midpoint of SK, prove that $AM = LN$.



(20) In the triangle $\triangle ABC, AB = AC$, point Dlies on AB, and point E lies on the extension of AC such that $BD = CE$. If DE intersects BC at G, prove that $DG = GE$.



Third Unit: Number Theory

Divisibility and Prime Factorization

Summary: This file is an introductory guide to the basic concepts of integer divisibility and prime factorization. We will review a set of practical divisibility rules, define prime and composite numbers, and establish the most important theorem in number theory: The Fundamental Theorem of Arithmetic. Finally, we will apply these concepts to show how factorization is a powerful tool for simplifying complex mathematical expressions.

1- Divisibility Rules

Before diving into factorization, it's useful to have a set of rules to quickly determine if a number is divisible by another. Formally, we say that an integer a divides an integer b, written as $a|b$, if there is an integer k such that $b=ak$.

- **Divisibility by 2:** A number is divisible by 2 if its last digit is even (0, 2, 4, 6, or 8).

  Example: The number 538 is divisible by 2 because its last digit is 8.
- **Divisibility by 3:** A number is divisible by 3 if the sum of its digits is divisible by 3.

  Example: The number 741 is divisible by 3 because the sum of its digits is 12 (7+4+1=12), and 12 is divisible by 3.
- **Divisibility by 4:** A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

  Example: The number 1,824 is divisible by 4 because the number 24 is divisible by 4.
- **Divisibility by 5:** A number is divisible by 5 if its last digit is 0 or 5.

  Example: The number 9,875 is divisible by 5 because its last digit is 5.

- **Divisibility by 6:** A number is divisible by 6 if it is divisible by both 2 and 3.

  Example: The number 432 is divisible by 6 because it is divisible by 2 (last digit is 2) and by 3 (sum of digits is 9).

- **Divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.

  Example: The number 2,853 is divisible by 9 because the sum of its digits is 18, and 18 is divisible by 9.

- **Divisibility by 10:** A number is divisible by 10 if its last digit is 0.

  Example: The number 12,340 is divisible by 10.

- **Divisibility by 11:** A number is divisible by 11 if the alternating sum (odd-placed digits minus even-placed digits) of its digits is divisible by 11.

  Example: Let's take the number 54,384. The alternating sum is $5 - 4 + 3 - 8 + 4 = 0$. Since 0 is divisible by 11, the number 54,384 is divisible by 11.

Example question: You have three numbers. Each one is divisible by one of the following numbers: 7, 9, or 11. Match each number with its correct divisor. **The numbers are:**

- 819,045
- 792,143
- 16,926

Solution: To solve this problem, we will apply the divisibility rules to each number to identify the correct divisor.

1. **Testing the number 819,045,** we will start by testing for divisibility by 9 because it is the easiest. **Rule for divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9. Sum of the digits of 819,045:

$$8 + 1 + 9 + 0 + 4 + 5 = 27.$$

   Since 27 is divisible by 9, the number 819,045 is divisible by 9. Therefore, the number 819,045 is divisible by 9.

2. **Testing the number 792,143.** This number looks complicated, so the divisibility rule for 11 is a good candidate to test. **Rule for divisibility by 11:** A number is divisible by 11 if the alternating sum of its digits is divisible by 11. **Application:** Let's calculate the alternating sum of the digits of 792,143 (starting from the right with subtraction, then addition): $3-4+1-2+9-7=-1+1-2+9-7=0-2+9-7=-2+2=0$. **Result:** Since 0 is divisible by 11, the number 792,143 is divisible by 11. Therefore, the number 792,143 is divisible by 11.
3. **The number 16,926,** since we have found the numbers divisible by 9 and 11, it is logical that this number must be the one divisible by 7.

2- Prime Numbers

All integers greater than 1 are either prime numbers or are composite, made up of a product of prime numbers.

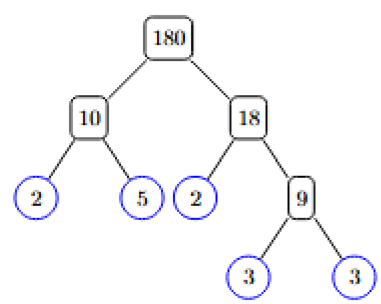
- **Definition (Prime Number):** A prime number is an integer greater than 1 whose only positive divisors are 1 and itself. Examples of prime numbers: (2, 3, 5, 7, 11, 47, 97).
- **Definition (Composite Number):** A composite number is an integer greater than 1 that is not prime. Examples of composite numbers: (4, 9, 15, 27, 180, 588).

The Fundamental Theorem of Arithmetic: Every integer greater than 1 is either a prime number or can be represented as a product of prime numbers, and this representation is unique, except for the order of the factors.

3- Methods of Prime Factorization

Factor Tree Method Example (Factoring the number 180):

- We break down the number 180 until we reach its prime factors. Note that 180 is the product of 10 and 18.
- We break down 18 into 9 and 2, and 10 into 2 and 5.
- We break down 9 into 3 and 3.
- Thus, the factorization of 180 is the product of the prime numbers $2^2 \times 3^2 \times 5$.



Repeated Division Method Example (Factoring the number 588):

We divide the number by the smallest prime that divides it until we reach 1. Then we count the divisors.

$588 \div 2 = 294$ $294 \div 2 = 147$ $147 \div 3 = 49$ $49 \div 7 = 7$ $7 \div 7 = 1$ So, $588 = 2^2 \times 3 \times 7^2$	It is sometimes easier to look at it in this way (*column factorization*)): 2 588 2 294 3 147 7 49 7 7 1

4-How Factorization Simplifies Calculations

The true power of factorization lies in its ability to reveal the internal structure of numbers, enabling us to simplify seemingly complex arithmetic operations like division, addition, and subtraction.

- **Simplifying Division (Fractions)** When dividing two large numbers (or simplifying a fraction), finding their common factors is the ideal method.

Example (Simplify the fraction):

$$\frac{396}{924}$$

We factor the numerator and the denominator using one of the methods to get:

$$396 = 2^2 \times 3^2 \times 11, \qquad 924 = 2^2 \times 3 \times 7 \times 11$$

So, when dividing the two numbers, we can cancel out the common prime factors to get:

$$\frac{396}{924} = \frac{2^2 \times 3^2 \times 11}{2^2 \times 3 \times 7 \times 11} = \frac{2^2}{2^2} \times \frac{3^2}{3} \times \frac{1}{7} \times \frac{11}{11} = \frac{3}{7}$$

Exercises:

(1) What is the result of the calculation $(20 + 18) \div (20 - 18)$?

$(A) 18$ $(B)\ 19$ $(C)\ 20$ $(D)\ 34$ $(E)\ 36$

(2) Which of the following numbers is closest to the result of the operation $\frac{17\times0.3\times20.16}{999}$?

$(A)\ 0.01$ $(B)\ 0.1$ $(C)\ 1$ $(D)\ 10$ $(E)\ 100$

(3) Mustafa knows that 1111×1111=1234321. What result does he get when calculating

$(A)\ 3456543$ $(B)\ 2345432$ $(C)\ 2234322$ $(D)\ 2468642$ $(E)\ 4321234$

(4) What number must replace the star (⋆) to make the following equation true: 2·18·14=6·⋆·7?

$(A)\ 8$ $(B)\ 9$ $(C)\ 10$ $(D)\ 12$ $(E)\ 15$

(5) Which of the following statements is correct?

$(A) \frac{4}{1} = 1.4$ $(B) \frac{5}{2} = 2.5$ $(C) \frac{6}{3} = 3.6$ $(D) \frac{7}{4} = 4.7$ $(E) \frac{8}{5} = 5.8$

(6) The sum of 5 consecutive integers is 10^{2018}. What is the middle number among them?

$(A) 10^{2013}$ $(B) 5^{2017}$ $(C) 10^{2017}$ $(D) 2^{2018}$ $(E)\ 2 \cdot 10^{2017}$

(7) Which of the following fractions is smaller than 2?

$(A) \frac{19}{8}$ $(B) \frac{20}{9}$ $(C) \frac{21}{10}$ $(D) \frac{22}{11}$ $(E) \frac{23}{12}$

(8) 2016 hours is how many weeks?

$(A)\ 6$ $(B)\ 8$ $(C)\ 10$ $(D)\ 12$ $(E)\ 16$

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(9) What is the sum of 25% of 1448 and 1448% of 25?

(A) 724 (B) 1448 (C) 2025 (D) 3024 (E) 5042

(10) Hamza has 20 Riyals, and each of his four brothers has 10 Riyals. How much must Hamza give to each brother so that everyone has the same amount of money?

(A) 2 (B) 4 (C) 5 (D) 8 (E) 10

(11) One-sixth of the audience in a children's theater are adults, the rest are children. Two-fifths ($\frac{2}{5}$) of the children are girls. What fraction of the total audience are boys?

$(A)\frac{1}{2}$ $(B)\frac{1}{3}$ $(C)\frac{1}{4}$ $(D)\frac{1}{5}$ $(E)\frac{2}{5}$

(12) Four cousins are 3, 8, 12, and 14 years old. Fatima is younger than Kholoud. The sum of Salwa's and Fatima's ages is divisible by 5, and so is the sum of Salwa's and Kholoud's ages. What is the age of Inas (the fourth cousin)?

(A)14 (B)12 (C)8 (D)3 (E)1448

(13) If you multiply the three digits of a three-digit number, you get 135. What result do you get by adding these three digits?

(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

(14) A rectangle has an area of 12 cm^2. The lengths of its sides are natural numbers. Which perimeter could the rectangle have?

(A) $20\ cm$ (B) $26\ cm$ (C) $28\ cm$ (D) $32\ cm$ (E) $48\ cm$

(15) Razan writes down three prime numbers that are all less than 100. She only uses the digits 1, 2, 3, 4, and 5, and in fact, she uses each digit exactly once. Which of the following prime numbers did Razan definitely write down?

(A) 2 (B) 5 (C) 31 (D) 41 (E) 53

(16) How many times does the summand 1448^2 appear under the root, if the following statement is correct?

$$\sqrt{1448^2 + 1448^2 + \cdots + 1448^2} = 1448^{10}$$

$(A)5$ $(B)8$ $(C)18$ $(D)1448^8$ $(E)1448^{18}$

(17) The following table is the multiplication table of the numbers 1 to 10. What is the sum of all 100 products in the complete table?

•	1	2	3	...	10
1	1	2	3	...	10
2	2	4	6	...	20
3	3	6	9	...	30
...	...	...	...	...	...
10	10	20	30	...	100

(A) 1000 (B) 2025 (C) 2500 (D) 3025 (E) 5500

(18) Ahmed writes three single-digit numbers on the board. Saud adds them and gets 15. Then he deletes one of the three numbers and replaces it with the number 3. Ahmed multiplies these three new numbers and gets 36. What are the numbers that Saud could have deleted?

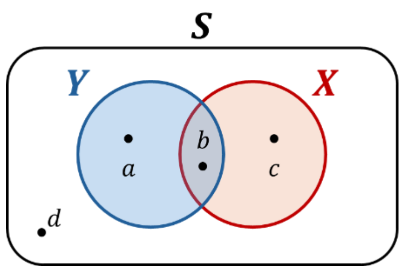
(A) 6 *or* 7 (B) 7 *or* 8 (C) 6 (D) 7 (E) 8

Forth Unit: Combinatorics

The Science of counting. Combinatorics

First: Counting using Venn forms

Venn forms are used to solve counting problems involving overlapping types in adjectives. Types are usually represented by overlapping circles, each of which has a number representing the number of elements of that type as shown in the figure:



a Achieves only the property Y

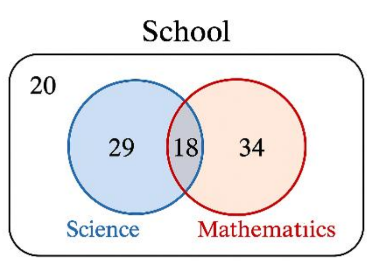
c Achieves only the property X

b Achieves both properties X and Y together

d does not achieve the property X and does not achieve the property Y (does not achieve nor property X or Y)

Example:

The following figure represents the numbers of students in a school who prefer a math class and who prefer a science class.



a) How many students prefer math? ....
b) How many students prefer science or math? ....
c) How many students prefer science and math together? ....
d) How many students prefer science only? ....
e) How many students do not prefer science? ....
f) How many students does the school have? ....

Solution

a) $18 + 34 = 52$
b) $29 + 18 + 34 = 81$
c) 18
d) 29
e) $18 + 34 + 20 = 72$
f) $18 + 34 + 29 + 20 = 101$

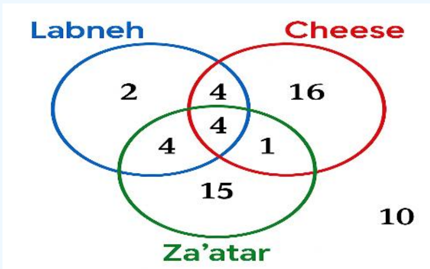
Exercises

(1) In one of our gifted classes, 35 students were offered to learn a foreign language, namely English and French. 15 students Choose an English language, and 5 students choose both languages. If we knew that any student chose to learn at least one of the two languages, how many students chose to learn only French?

(2) 40 People went on a trip, 18 of them prefer cheese pie and 15 of them prefer labneh pie, while 12 of them don't prefer either cheese or labneh. How many students prefer the two types together?

(3) A poll was conducted for a 50 students at a school to find out what type of sport they prefer, and the result was that 33 of them prefer soccer, 24 prefer basketball, and 11 prefer both. How many students don't prefer either two games?

(4) The figure shows the distribution of 60 students in a school, according to their preference for three types of pies. How many students prefer labneh?



(5) A factory manager conducted liked all three products from the factory, and found that 38 persons liked product A, 36 liked product B, 39 liked product C, 24 liked both products A and B, 20 liked both products C and A, 18 liked both products B and C, and finally 9 people liked all three products. How many people have liked only product C?

Second: Counting a list of numbers

It is easy to count numbers in the list $1,2,3,4,...,50$ and therefore it is called a simple list, hence it can be said that the list of numbers is simple if you meet the following conditions:

1- Starts with the number 1 2- has Consecutive contiguous numbers

Example:

How many numbers are in the list $3,6,9,...,327$

Solution:

The list is not simple, but it can be converted into a simple list by dividing all the numbers in the list by 3

$\div 3$	$3,6,9,...,327$	
	$1,2,3,...,109$	Simple list

so, the number of numbers is equal to 109

Example: How many numbers are in the list $23,28,33,...,548$

Solution: The list is not simple but can be converted into a simple list with the following steps

-3	$23,28,33,...,548$	
$\div 5$	$20,25,30,...,545$	
-3	$4,5,6,...,109$	
	$1,2,3,...,106$	Simple list

So, the number of numbers is equal to 106

Some lists can be converted to a simple list by performing an operation on all numbers in them, because operations on all numbers produce a list with the same number of numbers.

And we will see how to do that in the following exercises

Exercises

(6) How many numbers are in the following list $1,2,3,\ldots,1440$?

(7) How many numbers are in the following list $8,9,\ldots,2019$?

(8) How many numbers are in the following list $5,9,13,\ldots,505$?

(9) How many positive even numbers are less than 2025 ?

(10) How many numbers are in the following list: $\frac{3}{7},1,\frac{11}{7},\ldots,289$?

(11) How many numbers are there between 2023 , 63 so that it is a multiple of the number 3 ?

(12) How many positive integers are less than 600 so that it is a whole square?

(13) How many positive integers are less than 1447 so that it is multiple of both numbers 3 and 5?

(14) How many positive integers are less than 2025 which are multiples of the number 7 and not multiples of the number 5?

(15) How many positive integers are less than 500 which are multiples of the number 7 and not even?

Third: The Basic Principles of Counting

Let's start with this very simple example.

Example:

Mohammed and Ali went to a sportswear store that sells four different types of shoes and seven different types of sports gloves. Mohammed has enough money to buy one shoe and one glove, and Ali has enough money to buy only a shoe or just a glove. Question:

A) In how many ways can Mohammed buy a shoe and a glove?

Solution:

$$4 \cdot 7 = 28$$

B) In how many ways can Ali buy a shoe or a glove?

solution: $4 + 7 = 11$

Many counting problems use one of these two principles and are therefore called the two basic principles of counting. Let's get to know them and see creative ways to apply them to counting problems.

3-1 Multiplication Principle:

If the event A occurs in m different ways and the event B occurs in n different ways, and the two events are independent, then the two events can be done together in $m \cdot n$ ways

Example:

(1) How many ways can we arrange different books 5 on a shelf?

Solution:

First place	Second place	Third place	Fourth place	Fifth place
5 ways	4 ways	3 ways	2 ways	1 ways

When we choose a book to put in the first place at the beginning of the shelf, we will have 5 ways because we have 5 different books, and after we choose a book from them and put it in the first place, we will have 4 ways to choose the book that we will put in the second place, because the number of books decreased after putting one of them in the first place, and in the same way we will calculate the number of ways for each of the five places and then we use the multiplication principle to find the number of ways of ordering, so the number of ways = and this is equal to a method$5 \times 4 \times 3 \times 2 \times 1 = 120$

(2) How many ways can we choose different cards in color and different in value (the number written on them), if we have 52 a card divided into 4 colors, and the cards of each color are numbered in numbers from 1 to 13?

Solution: We need to choose a card of each color with a different number than the other color card

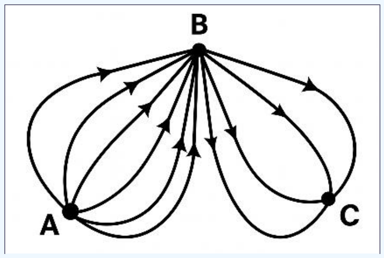
First Color	second color	third color	Fourth Color
13	12	11	10

We have 13 ways to choose a card of the first color, and when choosing a card of the second color, we must exclude the card that has the same number on the first card, so the number of ways will decrease and become 12 ways, and in the same way we have 11 ways to choose the third color card and 10 ways to choose the fourth color card, then we use the multiplication principle to find the number of ways, so the number of ways =

$13 \times 12 \times 11 \times 10$

Exercises

(16) There are three villages A, B, C in the country. You can move from village A to village B by 6 ways, and you can get from village B to village C by four ways, as shown in the figure. How many ways can you get from A to C?



(17) How many three-digit natural numbers are there?

(18) How many ways can 5 students be arranged in a class so that they are selected from 8 students?

(19) How many ways can you enter a complex from one door and exit through another if you know that the complex has ten doors?

(20) We want to arrange 4 daughters and 5 mothers in a row on the condition that the girls stay next to each other, how many ways can this be done?

3-2 Principle of Addition.

> If event A occurs in m different ways and event B occurs in n different ways, and the two events are not subtracted, then event A or B will occur in $m + n$ different ways.

We use this principle if solving a question requires dividing the problem into several contradictory cases, as in the previous example

Example:

If we have $\mathbf{5}$ different books on a shelf. How many ways can we arrange some (or all) books in a package? The package may contain (only one book) or more

Solution:

We have several cases:

The first case: The package consists of one book, so the number of ways =$\mathbf{5}$

The second case: The package consists of two books, so the number of methods

$= \mathbf{5 \times 4 = 20}$

The third case: The package consists of three books, so the number of methods

$=\mathbf{5 \times 4 \times 3 = 60}$

Fourth case: The package consists of four books, so the number of ways

$=\mathbf{5 \times 4 \times 3 \times 2 = 120}$

Fifth case: The package consists of five books, so the number of methods

$=\mathbf{5 \times 4 \times 3 \times 2 \times 1 = 120}$

Using the principle of combining states, the number of ways

$= \mathbf{5 + 20 + 60 + 120 + 120 = 325}$

Exercises

(21) How many ways can we choose a pen or a notebook from a set of 10 different notebooks and 7 different pens?

(22) Salma has 5 different types of teacups, 3 different types of serving plates, and 4 different types of teaspoons. How many ways can you choose two things of them?

(23) In the first grade of primary school, there is 12 girls, names of three of them start with the letter A. In how many ways can 12 girls be arranged in a row provided that only one of the girls starts her name with the letter A?

(24) The Talent School has three classes, the first class consists of 20 students, the second class consists of 13 students, and the third class consists of 8 students. In how many ways can two students be selected from two different classes to complete a task together?

(25) Traveling from city A to city B can be done by one of two land routes or by one of three air routes, and traveling from city B to city C can be done by one of four land routes or one of five air routes.

a) How many ways to travel from city A to city C through city B?
b) How many ways to travel from city A to city C through city B so you use two air routes?
c) How many ways to travel from city A to city C through city B so you use two road routes?
d) How many ways to travel from city A to city C through city B, so that you use one land route, one and one air route?

Solutions

Algebra Solutions

Integers and Their Properties:

Exercises:

(1)

$a)\ (-4)+9=5$
$b)\ -42\div 7=-6$
$c)\ (2)^5=32$
$d)\ (-4)^3=-64$
$e)\ (-5)^2=25$
$f)\ |0|=0$
$g)\ (-4)\times(-8)=32$
$h)\ -8-(-5)=-8+5=-3$
$i)\ -1-4+7=2$
$j)\ 2\times 4+6\times 5=8+30=38$
$k)\ |-6|=6$
$m)|-3|-|-7|=3-7=-4$

(2)

$a)\ a^2\times a^5=a^{2+5}=a^7$
$b)\ x^7\div x^3=x^{7-3}=x^4$
$c)\ (a^3)^4=a^{3\times 4}=a^{12}$
$d)\ (x^2)^3\times(x^4)^5=x^{2\times 3}\times x^{4\times 5}=x^6\times x^{20}=x^{26}$
$e)\ \frac{a^3\times a^7}{a^2\times a^6}=\frac{a^{3+7}}{a^{2+6}}=\frac{a^{10}}{a^8}=a^{10-8}=a^2$
$f)\ \frac{a^4\times a^5}{a^3\times a^6}=\frac{a^{4+5}}{a^{3+6}}=\frac{a^9}{a^9}=a^{9-9}=a^0=1$

(3)

$a)\ (-7)+(-12)-(-14)-(-15)-(-18)-(-38)$
$=-7-12+14+15+18+38$
$=66$
$b)\ (-5)^2+|-6|-(-1)^{1447}$
$=25+6+1$
$=32$

Challenge Problems:

(1)

$$-1-(-1)^1-(-1)^2-(-1)^3-\cdots-(-1)^{99}-(-1)^{100}$$
$$=-1+\underbrace{(1-1+1-1+\cdots+1-1)}_{100\ terms}$$
$$=-1+0$$
$$=-1$$

(2)

$$1234\times 9999$$
$$=1234(10000-1)$$
$$=1234\times 10000-1234\times 1$$
$$=12340000-1234$$
$$12338766$$

(3)

$$5\{(2a-3)-[7(4a-1)-20]\}-(3-8a)$$
$$=5\{2a-3-28a+7+20\}-3+8a$$
$$=5\{-26a+24\}-3+8a$$
$$=-130a+120-3+8a$$
$$=-122a+117$$

(4)

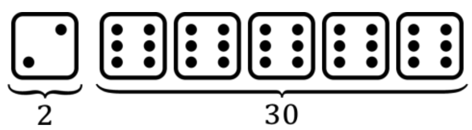
The number of orbits completed by a satellite in one week equals the number of hours in a week divided by 7.

$$=(24\times 7)\div 7=\frac{24\times 7}{7}=24$$

(5)

To obtain the smallest number on one of the dice, the other five dice must show the **largest possible number**.

Therefore, this occurs when each of the other five dice shows the number **6**, and then the **smallest number** appears on the sixth die, which is **2**.



(6)

$$1 - 2 + 3 - 4 + \cdots - 100 + 101$$
$$= (1 - 2) + (3 - 4) + \cdots + (99 - 100) + 101$$
$$= \underbrace{-1 - 1 - 1 \ldots - 1}_{50\ times} + 101$$
$$= -50 + 101$$
$$= 51$$

Rational Numbers $\mathbb{Q}$

Exercises:

(1)

$$a)\ \frac{2}{5}+\frac{1}{6}=\frac{12+5}{30}=\frac{17}{30}$$

$$b)\ \frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\frac{1}{6}$$

$$c)\ \frac{-3}{8}\times\frac{4}{9}=\frac{-3}{9}\times\frac{4}{8}=\frac{-1}{3}\times\frac{1}{2}=\frac{-1}{6}$$

$$d)\ \frac{-3}{5}-\left(-\frac{1}{2}\right)=\frac{-3}{5}+\frac{1}{2}=\frac{-6+5}{10}=\frac{-1}{10}$$

$$e)\ 1\div 3\div 4\div 5=1\times\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}=\frac{1}{60}$$

$$f)\ \frac{1}{9}-\frac{1}{10}=\frac{10-9}{90}=\frac{1}{90}$$

$$g)\ (-3)\div 4\times 6\div(-5)=-3\times\frac{1}{4}\times 6\times\left(-\frac{1}{5}\right)=\frac{18}{20}=\frac{9}{10}$$

$$h)\ \frac{-6}{35}\div\frac{2}{7}=\frac{-6}{35}\times\frac{7}{2}=\frac{-6}{2}\times\frac{7}{35}=\frac{-3}{1}\times\frac{1}{5}=\frac{-3}{5}$$

$$i)\ \left(\frac{-1}{2}\right)^5=\frac{(-1)^5}{2^5}=\frac{-1}{32}$$

(2)

$$a)\ 0.\overline{2}=\frac{2}{9}$$

$$b)\ 0.\overline{37}=\frac{37}{99}$$

$$c)\ 1.8\overline{23}=\frac{18}{10}+\frac{1}{10}\times 0.\overline{23}$$

$$=\frac{18}{10}+\frac{1}{10}\times\frac{23}{99}$$

$$=\frac{18}{10}+\frac{23}{990}$$

$$=\frac{1805}{990}$$

$$=\frac{361}{198}$$

(3)

Yes, the claim is correct.

When we divide the number $\frac{a}{b}$ by $\frac{c}{d}$, the result is equal to $\frac{a}{b} \times \frac{d}{c}$,

which means we multiply by the multiplicative inverse of $\frac{c}{d}$.

(4)

Saleh's claim is correct. Let's take any two rational numbers, for example:

$$\frac{2}{7}, \frac{3}{7}$$

They may look close to each other, and it might seem that there are no rational numbers between them.

However, this is not true. If we rewrite them in an equivalent form, for instance:

$$\frac{20}{70}, \frac{30}{70}$$

we can clearly find numbers between them, such as:

$$\frac{21}{70}, \frac{22}{70}, \dots, \frac{29}{70}$$

And if we express them again with larger equivalent denominators, for example:

$$\frac{200}{700}, \frac{300}{700}$$

We will find even more numbers between them.

Therefore, there are infinitely many rational numbers between any two rational numbers.

(5)

$$-\frac{1}{3}, -0.3, -0.23$$

Challenge Problems:

(1)

To begin, notice that:

$$\frac{1}{6}=\frac{5}{30} \text{ and } \frac{1}{3}=\frac{10}{30}$$

Therefore, any fraction between them can be written in the form:

$$\frac{k}{30}$$

where k is an even number.

If we let k take the values **6** and **8**, we obtain only two fractions.

(2)

We know that

$$\frac{1}{4}=0.25$$

Therefore, N is equal to one-fourth of 8, and hence

$$N=2$$

Thus, Khaled will obtain

$$16$$

(3)

In total, Haitham has completed the first half of the journey, plus $\frac{3}{5}$ of the second half.

Therefore, the total distance he has covered is:

$$\frac{1}{2}+\frac{3}{5}\times\frac{1}{2}=\frac{1}{2}+\frac{3}{10}=\frac{8}{10}=\frac{4}{5}$$

So, Haitham has covered $\frac{4}{5}$ of his journey.

(4)

$$\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{9}-\frac{1}{10}\right)$$
$$=1+\left(-\frac{1}{2}+\frac{1}{2}\right)+\left(-\frac{1}{3}+\frac{1}{3}\right)+\cdots+\left(-\frac{1}{9}+\frac{1}{9}\right)-\frac{1}{10}$$
$$=1-\frac{1}{10}$$
$$=\frac{9}{10}$$

(5)

$$\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\ldots\cdot\frac{79}{80}$$
$$=\frac{1}{80}$$

(6)

A		7						e	d	c	7	b	4

Since the sum of $\boldsymbol{b}$, $\mathbf{7}$, and 4equals $\mathbf{20}$, it follows that $\boldsymbol{b} = \mathbf{9}$.

Next, $\mathbf{7} + \mathbf{9} + \boldsymbol{c} = \mathbf{20}$, so $\boldsymbol{c} = \mathbf{4}$.

Continuing this pattern to the left, we find $\boldsymbol{d} = \mathbf{9}$ and $\boldsymbol{e} = \mathbf{7}$.

By maintaining this sequence, we finally reach $\boldsymbol{A} = \mathbf{9}$.

(7)

Leila is now **32 years old**, and the **sum of her two daughters' ages is 5 years**.

This means that the difference between Leila's age and the **sum of her daughters' ages** is **27 years**.

Each year, Leila's age increases by one year, while the sum of her daughters' ages increases by two years, and the difference between them decreases by one year.

Since the difference is now **27** years, it will take **27 years** for the difference to become zero.

At that time, Leila's age will be $\mathbf{32} + \mathbf{27} = \mathbf{59}$ years.

(8)

The maximum number of matches that any team can play is **2**, since each team can play **at most once per year**, and there are **only three teams**.

Therefore, **Team B** and **Team C** must each have played **only one match**.

As for **Team A**, it must have played **two matches**, because if it had played **zero matches**, it would have scored **zero goals**, while according to the problem statement, it has scored either **5 or 3 goals**.

Since each recorded number must either increase or decrease by **1**, it follows that **Team A** has:

1 win, **1 draw**, and **0 losses** (since the total number of matches played is **2**).

Because **Team A** won one match, **Team B** must have lost one match (since **Team B** originally had **0** losses, and **Team C** cannot have lost **2** matches).

Therefore, **Team B** has **1 loss** and **0 wins or draws**.

We can now deduce that **Team C** drew with **Team A**; thus, **Team C** has **1 draw**, **0 wins** and **losses**.

Since **Team C's only match** ended in a draw, its **goals for** and **goals against** must be **equal**.

Hence, **Team C** scored **2 goals** and conceded **2 goals**.

The only match played by **Team B** ended in a **loss**, so the number of **goals for Team B** must be **less** than the number of **goals against**.

This is satisfied when **Team B** scored **1 goal** and conceded **3 goals**.

Since both **Team B** and **Team C** played **only against Team A**, the **total number of goals scored by Team A** equals the **sum of the goals conceded by Teams B and C**.

Therefore, **Team A** scored a total of **5 goals**.

Similarly, the **total number of goals conceded by Team A** equals the **sum of the goals scored by Teams B and C**.

Thus, **Team A** conceded **3 goals**.

Hence, the **correct table** is as follows:

Team	Played	Won	Drawn	Lost	Goals For	Goals Against
A	2	1	1	0	5	3
B	1	0	0	1	1	3
C	1	0	1	0	2	2

Linear Equations in One Variable:

Exercises:

(1)

$a)\ x = 1$ $b)\ x = 11$ $c)\ x = 7$

$d) x = 18$ $e)\ x = 10$ $f)\ x = 5$

$g)\ x = 8$

(2)

$x = 1$

(3)

$x = 3$

Word Problems Solved Using Linear Equations:

(1)

Let the number be x . Then,

$$\frac{7}{100}x = 56$$

Therefore $x = 800$

(2)

Let the numbers be $n, n + 1, n + 2, and\ n + 3.$

Then, we have

$$4n + 6 = 50$$

Therefore $n = 11$

Hence, the largest number is **14**.

(3)

Let the two numbers be $2x\ and\ 3x$.

Then,

$$3x - 2x = 14$$

Hence,

$$x = 14$$

Therefore, the smaller number is **28**.

(4)

Let the original price be x.

After a discount, the price becomes **65%** of the original price.

Hence,

$$\frac{65}{100}x = 1300$$

Therefore, $x = 2000$ riyals.

(5)

Let the purchase price be x.

The selling price is **115%** of the purchase price.

Hence,

$$\frac{115}{100}x = 46000$$

Therefore,

$$x = 40000 \text{ riyals.}$$

Challenge Problems:

(1)

Let the length of the train be x meters.

Since it takes **60 seconds** to completely pass through a tunnel that is **120 meters** long, starting from the moment the train enters until it fully exits, the constant speed of the train is:

$$\frac{120 + x}{60}$$

The same train takes **20 seconds** to completely pass a signal post at the same speed, which equals:

$$\frac{x}{20}$$

Hence,

$$\frac{120 + x}{60} = \frac{x}{20}$$

Solving this equation gives $x = 60$,

which means that the length of the train is **60 meters**.

(2)

Let the number of red balls at the beginning be n, then the number of white balls at the beginning is $4n$.

After replacing **2** white balls with **7** red ones, the number of red balls becomes $(n + 7)$,

and the number of white balls becomes $(4n - 2)$.

so we can write the equation:

$$\frac{n+7}{4n-2}=\frac{2}{3}$$

Simplifying gives:

$$3(n+7)=2(4n-2)$$
$$3n+21=8n-4$$
$$n=5$$

Therefore, the total number of balls at the beginning is:

$$n+4n=5n=25$$

The total number of balls at the end is:

$$(n+7)+(4n-2)=5n+5=30$$

Hence, the required ratio of the total numbers is:

$$30:25=6:5$$

(3)

Let the base year be the year when **Laila** was **20 years old**.

Suppose that after $\boldsymbol{x}$ **years**, Laila's age will be equal to the **sum of the ages of her three children**.

At that time, Laila's age will be $\boldsymbol{x}+20$ years,

and the ages of her three children will be $\boldsymbol{x}$, $\boldsymbol{x}-2$, and $\boldsymbol{x}-4$ years respectively.

Hence, we can form the equation:

$$x+20=x+(x-2)+(x-4)$$

Simplifying:

$$x+20=3x-6\Rightarrow 26=2x\Rightarrow x=13$$

Therefore, Laila's age at that time will be: $20+13=33$ years.

(4)

Let there be $\mathbf{3x}$ **apples** and $\mathbf{8x}$ **oranges** in the basket at the beginning.

After removing one apple, the number of apples in the basket becomes $\mathbf{3x-1}$,

while the number of oranges remains $\mathbf{8x}$.

The ratio of apples to oranges is now **1 : 3**,

so we can form the equation:

$$\frac{3x-1}{8x}=\frac{1}{3}$$

Solving for $\boldsymbol{x}$ gives:

$$x=3$$

Therefore, the number of oranges in the basket is:

$$8x=8(3)=24$$

Hence, there are **24 oranges** in the basket.

(5)

The sum of six numbers is **24**,

since $4\times 6=24$.

The sum of seven numbers is **35**,

since $5\times 7=35$.

Therefore, the seventh number is: $35-24=11$

(6)

Since **17** is less than **20**, and **481** is less than **500**,

the product $20\times 500=10{,}000$.

Therefore, the actual cost of the balls is **less than 10,000**.

(7)

It is not possible.

If one of the two numbers is even, then the product of the two numbers will be even.

When this product is multiplied by the sum of the numbers (whether that sum is even or odd),

the final product will still be even.

However, if both numbers are odd, their sum will be even, and their product will be odd.

When an odd number is multiplied by an even number, the result is even.

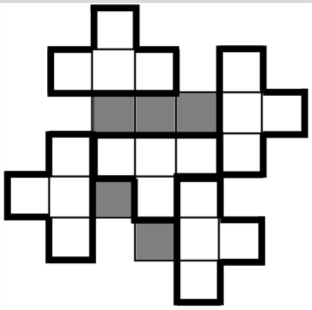
Therefore, in every possible case, the final result will be **even**,

whereas **20042401** is **odd**.

(8)

$$\begin{array}{r} 983 \\ +\ \ 75 \\ 4 \\ \hline 1062 \end{array}$$

(9)



(10)

The third cow produces half the amount of milk.

Therefore, the second cow produces one quarter of the milk,

which equals 2 liters more than what the first cow produces.

So, $2 + 2 = 4$ liters form one quarter of the milk.

This means the second cow produces $4 + 2 = 6$ liters,

and the third cow produces 8 liters of milk.

In conclusion:

the first cow produces **2 liters**,

the second cow **6 liters**,

and the third cow **8 liters**,

giving a total of **16 liters** of milk.

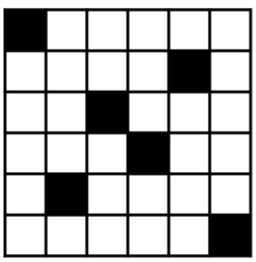
(11)

To ensure that there is no white strip of size 1×6, no row or column should be completely white.

By placing black squares as shown in the grid,

we also make sure that there is no white square block of size 3×3 after coloring.

One possible solution is shown below:



(12)

Let's first determine the possible values of **O**.

Since the number **3O** has a units digit of **O**, the possible values for **O** are **0** or **5**.

– If **O = 0**, there is no carryover to the tens' column.

Therefore, we need to determine the value of **M**.

The number **3M** must also have a unit's digit **M**, which means the possible values for **M** are **0** or **5**.

However, since **O** and **M** represent different digits, **M = 5**.

In this case, we have a carryover of **1** into the third column, meaning:

$$3J + 1 = I$$

Considering that **J ≠ 0**, the possible solutions are:

$$I = 4,\ J = 1 \text{ or } I = 7,\ J = 2.$$

– If **O = 5**, then **3O = 15**, producing a carryover of **1** into the middle column.

This requires:

$$3M + 1 = M + 10 \text{ or } 3M + 1 = M + 20 \text{ or } 3M + 1 = 2M + 9 \text{ or } 3M + 1 = 2M + 19.$$

It is clear that no valid integer value for **M** satisfies these cases.

Hence, the only possible solutions are:

$$JMO = 150,\ IMO = 450 \text{ and } JMO = 250,\ IMO = 750.$$

Points, Lines, and Angles

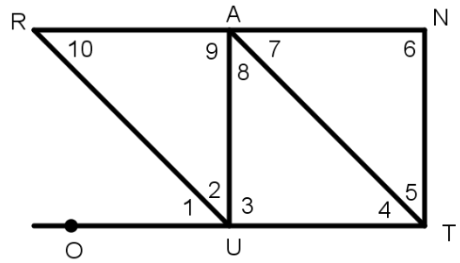
Exercises

(1)

$a)\ \angle 1 \cong \angle 4 \Longrightarrow RU||AT.$

$b)\ m\angle 2 \cong m\angle 10 \Longrightarrow$ no parallel lines needed

$d)\ \angle 5 \cong \angle 7 \Longrightarrow$ no parallel lines needed



$d)\ \angle 5 \cong \angle 8 \Longrightarrow AU||NT.$

$e)\ m\angle 6 = m\angle 9 = 90° \Longrightarrow AU||NT.$

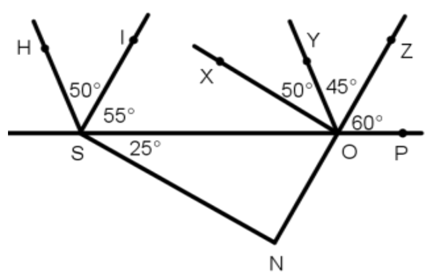
$f)\ m\angle 6 = m\angle 3 = 90° \Longrightarrow$ no parallel lines needed

$g)\ m\angle 7 = m\angle 10 = m\angle 1 \ \Longrightarrow RU||AT\ , RA||OU$

(2)

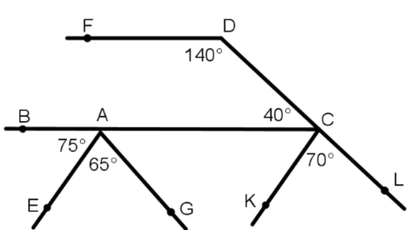
$\angle HSO = \angle YOP = 105 \Longrightarrow HS \parallel YO$

$\angle XOS = 180 - (60 + 45 + 50) = 25 = \angle NSO \Longrightarrow NS \parallel XO$



$\angle FDC + \angle DCA = 180 \Longrightarrow FD \parallel AC$

$\angle CAG = 180 - (75 + 65) = 40 = \angle DCA \ \Longrightarrow DC \parallel AG$



(3)

a) $x = 180 - 120 = 60°$

b) $x = 60°$, $y = 61°$

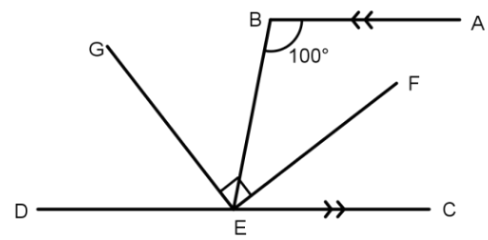
(4)

$\angle BEC = 180 - 100 = 80°$

$\angle BEF = \angle CEF = 80 \div 2 = 40°$

$\angle BEG = 90 - 40 = 50°$

$\angle DEG = 180 - (90 + 40) = 50°$



(5)

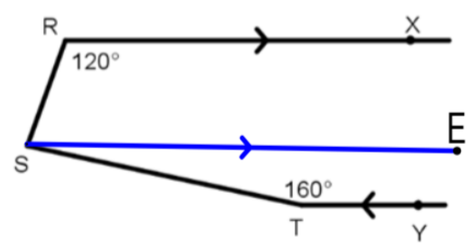
$\angle d = 180 - \angle a = 180 - 120 = 60°$

(6)

$\angle RSE = 180 - 120 = 60°$

$\angle TSE = 180 - 160 = 20°$

$\angle TSR = 60 + 20 = 80°$

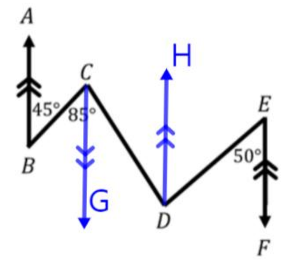


(7)

$\angle BCG = 45°$, $\angle DCG = 85 - 45 = 40°$

$\angle CDH = 40°$, $\angle EDH = 50°$

$\angle CDE = 50 + 40 = 90°$

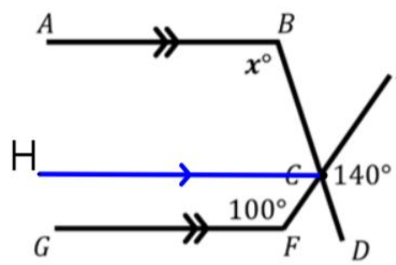


(8)

$\angle HCF = 180 - 100 = 80°$

$\angle HCB = 140 - 80 = 60°$

$x = 180 - 60 = 120°$



Triangles:

Exercises:

(1)

$a)\ m\angle 6 = 40 + 60 = 100°$

$b)\ m\angle 5 = 45 + 70 = 115°$

$c)\ m\angle 4 = 50 + 65 = 115°$

$d)\ m\angle 3 = 135 - 60 = 75°$

$e)\ m\angle 3 = 120 - 40 = 80°$

$f)\ m\angle 4 + m\angle 5 + m\angle 6 = 360°$

(2)

$40 + 80 + \angle A = x + 50 + \angle A \Longrightarrow x = 80 + 40 - 50 = 70°$

(3)

$\angle ABC = 75 - 50 = 25°$

$x = 180 - (110 + 25) = 45°$

(4)

$x = 360 - (107 + 153) = 100°$

(5)

$x = 180 - (20 + 60) = 100°$

(6)

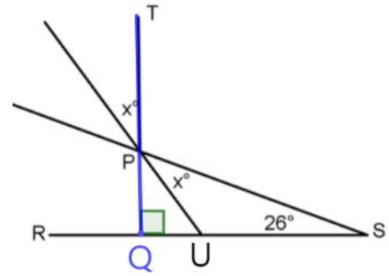
$\angle QPR = 60°\ , \angle PQT = 60 \div 3 = 20°$

$\angle QTP = 180 - (60 + 20) = 100°$

(7)

$\angle QPU = x$

$2x + 26 = 90 \Longrightarrow x = 32°$



(8)

$\angle UTV = 180 - 110 = 70°$

$\angle TVU = \angle TUV = (180 - 70) \div 2 = 55°$

$\angle QUV = 180 - 55 = 125°$

(9)

$\angle EDC = \angle DCB = 40 \div 2 = 20°$

$\angle BDC = 180 - (20 + 70) = 90°$

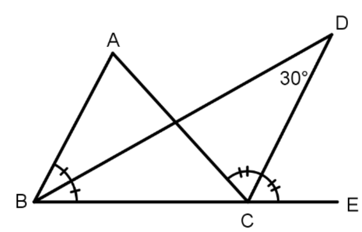
(10)

$\angle ECD = \angle DCA = y$

$\angle ABD = \angle DBC = x$

$y - x = 30$

$\angle A = 2y - 2x = 2(y - x) = 2 \times 30 = 60°$

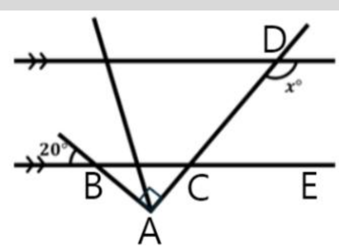


(11)

$\angle ABC = 20°$

$\angle ECD = \angle ACB = 90 - 20 = 70°$

$x = 180 - 70 = 110°$



Quadrilaterals:

Exercises:

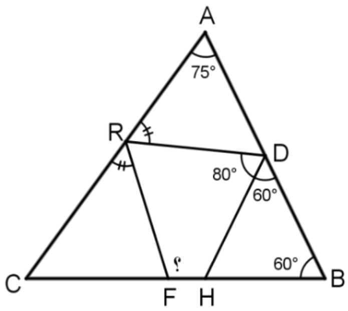
(1)

$\angle FHD = 60 + 60 = 120°$
$\angle ADR = 180 - (60 + 80) = 40°$

$\angle CRF = \angle RDA = 180 - (75 + 40) = 65°$

$\angle FRD = 180 - (65 + 65) = 50°$

$\angle HFR = 360 - (120 + 80 + 50) = 110°$

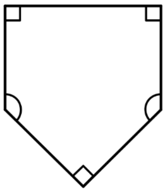


(2)

$3 \times 90° + 2x = (5 - 2) \times 180°$

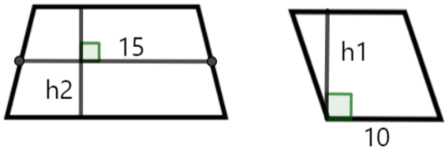
$2x = 270°$

$x = 135°$



(3)

$15 \times h_2 = 10 \times h_1 \Longrightarrow \frac{h_2}{h_1} = \frac{10}{15} = \frac{2}{3}$

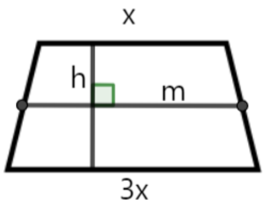


(4)

$h = m = \frac{x + 3x}{2} = 2x$

$A = h \times m = (2x)(2x) = 100$

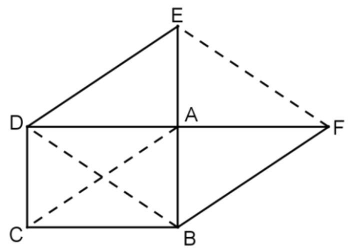
$\Longrightarrow 4x^2 = 100 \Longrightarrow x^2 = 25 \Longrightarrow x = 5\ , 3x = 15$



(5)

Notice that $DBFE$ is a parallelogram, which means that:

$$BF = CA = DE \Longrightarrow BF = DE\ , BF \parallel CA \parallel DE \Longrightarrow BF \parallel DE$$



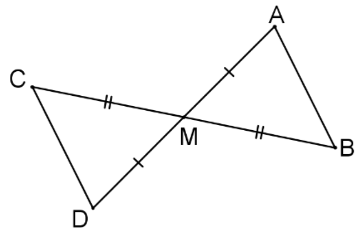
Triangle Congruence:

Exercise solutions:

Exercise 1:

In triangles AMB, DMC, we have:

$$\begin{cases} \boldsymbol{MA = MD} \\ \boldsymbol{MB = MC} \\ \boldsymbol{\angle AMB \cong \angle DMC} \end{cases}$$



Therefore, we have AMB, DMC are congruent and that gives the required results:

1) $AB = CD$

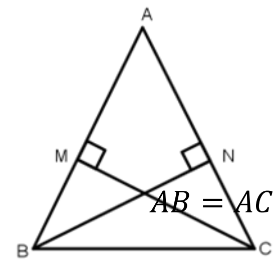
2) $m\angle A = m\angle D$ but they are interior angles. Thus, $AB \parallel CD$.

(1-15)

Ex. No.	Congruence type	Triangle
1	$A.S.A$	NPY
2	None	None
3	$S.S.S$	CKA
4	None	None
5	None	None
6	None	None
7	$S.A.S$	PQC

Ex. No.	Congruence type	Triangle
8	None	None
9	$A.S.A$	AGC
10	$A.S.A$	CDA
11	$A.S.A$	PST
12	None	None
13	None	None
14	$S.A.S$	CDA
15	$A.S.A$	MNC

(16)



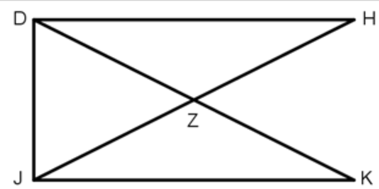
Notice that:

$m\angle AMC = m\angle ANB = 90°$ and $\angle A$ is a common angle for both triangles. Thus, $\angle NBA \cong \angle ACM$, but it is given that: $AB = AC$

Therefore, from $(A.S.A)$ we have,

$$\Delta ABN \cong \Delta ACM$$

(17)



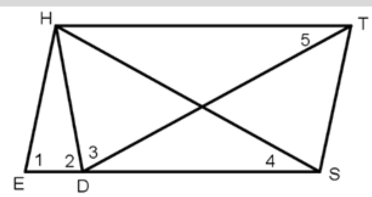
From the given conditions, notice that:

$\Delta JDH, \Delta DJK$

Which gives that:

$\angle H = \angle K$

(18)



We will try to find the conditions for the congruence

$$\Delta HES, \Delta HDT$$

We are given $ES = DT$,$\angle 1 = \angle 3$,

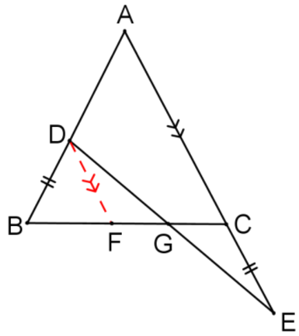
In ΔHED, we have $HD = HD$ since ,$\angle 1 = \angle 2$

Thus $\Delta HES, \Delta HDT$ are congruent and that gives that ,$\angle 4 = \angle 5$.

(19)

It is easy to show the congruence of LMK, AMF using the information given In the question.

(20)



From point D, we draw $DF \parallel AE$ as shown.

Notice that: $\angle FDG = \angle CEG$, $\angle DGF = \angle EGC$ by parallelism.

But since $\angle BFD = \angle BCA = \angle DBF$. Hence,

$$DF = DB = CE \Rightarrow \Delta DFG \cong \Delta ECG \Rightarrow DG = GE$$

Number Theory Solutions

(1)

First, we calculate what is inside the parentheses:

$20+18=38$, $20-18=2$

Next, we perform the division: $38 \div 2=19$.

The correct answer is 19.

(2)

This problem relies on estimation to simplify calculations. We round the numbers to the nearest easy value:

$17 \approx 20$, $20.16 \approx 20$, $999 \approx 1000$.

Now the approximate calculation becomes:

$$\frac{20 \times 0.3 \times 20}{1000} = \frac{120}{1000} = 0.12$$

The result 0.12 is closest to 0.1. The correct answer is 0.1.

(3)

We can rewrite the problem using factorization.

Note that $2222=2\times1111$. So, the operation becomes:

$$1111 \times (2 \times 1111) = (1111 \times 1111) \times 2$$

Since we know the value of (1111×1111) from the question, we substitute it:
$1234321 \times 2 = 2468642$

The correct answer is 2468642.

(4)

We use prime factorization to simplify both sides of the equation, just as explained in the lesson.

Left side: $2 \cdot (2 \cdot 3^2) \cdot (2 \cdot 7) = 2^3 \cdot 3^2 \cdot 7$ Right side: $(2 \cdot 3) \cdot\star\cdot 7$

By equating the two sides, we can find the value of the star by cancellation:

$$\star = \frac{2^3 \cdot 3^2 \cdot 7}{2 \cdot 3 \cdot 7} = 2^{3-1} \cdot 3^{2-1} \cdot 7^{1-1} = 2^2 \cdot 3^1 = 12$$

The correct answer is 12.

(5)

We calculate the value of each fraction to check the statement's validity:

(A) $\frac{4}{1} = 1.4$, which is not equal to 1.4.

(B) $\frac{5}{2} = 2.5$. This statement is correct.

(C) $\frac{6}{3} = 3.6$, which is not equal to 3.6.

(D) $\frac{7}{4} = 4.7$, which is not equal to 4.7.

(E) $\frac{8}{5} = 5.8$, which is not equal to 5.8.

The correct answer is 2.5.

(6)

The sum of 5 consecutive integers is always 5 times the middle number.

If the middle number is n, the sum is 5n.

$5n = 10^{2018}$, then $n = \frac{10^{2018}}{5}$.

To simplify this fraction, we can factor the numerator: $n = \frac{10 \times 10^{2017}}{5} = (\frac{10}{5}) \times 10^{2017} = 2 \times 10^{2017}$

The correct answer is $2 \cdot 10^{2017}$.

(7)

To see if a fraction is smaller than 2, we compare the numerator to twice the denominator.

(A) 19>2×8=16.

(B) 20>2×9=18.

(C) 21>2×10=20.

(D) 22=2×11=22.

(E) 23<2×12=24. This fraction is smaller than 2.

The correct answer is $\frac{23}{12}$.

(8)

The number of hours in one week is 7 days×24 hours/day=168 hours.

We need to calculate $\frac{2016}{24\times7}$.

We can simplify this fraction using divisibility rules (instead of calculating 24×7).

The correct answer is 12.

(9)

We note that the two operations are identical due to the commutative property of multiplication:

25% of 1448 is $\frac{25}{100} \times 1448$.

1448% of 25 is $\frac{1448}{100} \times 25 = \frac{25}{100} \times 1448$.

Therefore, the required sum is

$$2 \times 1448 \times \frac{25}{100} = \frac{2}{4} \times 1448 = 724$$

The correct answer is 724.

(10)

Total amount: 20+(4×10)=60 Riyals.

Number of people: 5.

Equal share: 60÷5=12 Riyals per person.

Each brother needs 12 Riyals. Therefore, Hamza must give 2 Riyals to each brother.

The correct answer is 2.

(11)

Fraction of children = $\frac{5}{6}$.

Fraction of boys = (Fraction of children) * (Fraction of boys among children) = $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$.

The correct answer is $\frac{1}{2}$.

(12)

The ages are {3, 8, 12, 14}. Since "Salwa's" age is common in two sums that are multiples of 5, we look for a number that forms a sum divisible by 5 with two other numbers.
3+12=15, and 8+12=20.

Therefore, Salwa's age is 12.

Fatima's and Khaloud's ages are 3 and 8. The ages used are 3, 8, and 12. The remaining age for Inas is 14.

The correct answer is 14.

(13)

We need to find three digits (from 1 to 9) whose product is 135. The best way is the prime factorization of 135.

$$135 = 5 \times 27 = 5 \times 3 \times 9 = 5 \times 3 \times 3 \times 3.$$

The prime factors are {3, 3, 3, 5}.

We must group them to form three digits. One of the numbers cannot be 3×5=15 because it is not a single digit. We can group two factors: 3×3=9. Thus, the three digits are 9 and the two remaining factors 3 and 5. The digits are {3, 5, 9}. Check: 3×5×9=15×9=135. This is correct. Now, we add these digits: 3+5+9=17.

The correct answer is 17.

(14)

Area = Length x Width = 12. Since the sides are natural numbers, we are looking for the factor pairs of 12. The factor pairs of 12 are:

- 1 and 12
- 2 and 6
- 3 and 4

If the dimensions are 1 and 12: Perimeter = 2×(1+12)=2×13=26 cm. If the dimensions are 2 and 6: Perimeter = 2×(2+6)=2×8=16 cm. If the dimensions are 3 and 4: Perimeter = 2×(3+4)=2×7=14 cm. Among the given options, the only possible perimeter is 26 cm.

The correct answer is 26 cm.

(15)

The available digits are {1, 2, 3, 4, 5}. They must be used to form three prime numbers less than 100. Since we have 5 digits, the primes must be in the form: one one-digit number and two two-digit numbers (because 1+2+2=5). The one-digit primes that can be formed are 2, 3, 5. Let's try the cases:

Case 1: One number is 2. The remaining digits are {1, 3, 4, 5}. We need to form two two-digit primes. Let's look for pairs: If we take 41, the remaining are {3, 5}. We can form the prime 53. The set is {2, 41, 53}. This is a valid set.

Case 2: One number is 3. The remaining digits are {1, 2, 4, 5}. We need to form two two-digit primes. If we take 41, the remaining are {2, 5}. We cannot form a prime. If we take 51 (not prime). With 2, 4, 1, 5, we cannot form two primes.

Case 3: One number is 5. The remaining digits are {1, 2, 3, 4}. If we take 41, the remaining are {2, 3}. We can form the prime 23. The set is {5, 41, 23}. This is a valid set. We found two possible sets of prime numbers: {2, 41, 53} and {5, 23, 41}. The common prime number in both possible sets is 41. Therefore, this is the number that Razan definitely wrote.

The correct answer is 41.

(16)

Let's assume the number 1448^2 is repeated n times. We can write the equation as:

$$\sqrt{n \times 1448^2} = 1448^{10}$$

We can simplify what's under the root:

$$1448 \times \sqrt{n} = 1448^{10}$$

Now, we divide both sides by 1448:

$$\sqrt{n} = 1448^9$$

To find n, we square both sides:

$$n = 1448^{18}$$

So, the number 1448^2 is repeated 1448^{18} times.

The correct answer is 1448^{18}.

(17)

The sum of all products in the table can be written as:

$S = (1 \times 1 + 1 \times 2 + \cdots + 1 \times 10) + (2 \times 1 + \cdots + 2 \times 10) + \cdots + (10 \times 1 + \cdots + 10 \times 10)$ We can take a common factor from each row:

$$S = 1(1 + 2 + \cdots + 10) + 2(1 + 2 + \cdots + 10) + \cdots + 10(1 + 2 + \cdots + 10)$$

Now, we take the parentheses (1+2+⋯+10) as a common factor:

$$S = (1 + 2 + \cdots + 10) \times (1 + 2 + \cdots + 10)$$

The sum of numbers from 1 to 10 is 55. Therefore, the total sum is S=55×55=3025.

The correct answer is 3025.

(18)

Let the original numbers written by Ahmed be x, y, z.

First given: $x + y + z = 15$. Saud deleted one of the numbers (let's say z) and replaced it with 3. The new numbers are $x, y, 3$.

Second given: $x \cdot y \cdot 3 = 36$. From the second given, we can find x and y from the equation $x \cdot y = 12$. We are looking for two single-digit numbers whose product is 12.

The possible pairs are: 2 and 6, 3 and 4. Now, we use the first given to find the deleted number z.

Case 1: If x=2, y=6. 2+6+z=15⟹8+z=15⟹z=7. The deleted number could be 7. The original numbers were {2, 6, 7}.

Case 2: If x=3, y=4. 3+4+z=15⟹7+z=15⟹z=8. The deleted number could be 8. The original numbers were {3, 4, 8}.

So, the number that Saud deleted could be either 7 or 8.

The correct answer is 7 or 8.

Combinatorics Solutions

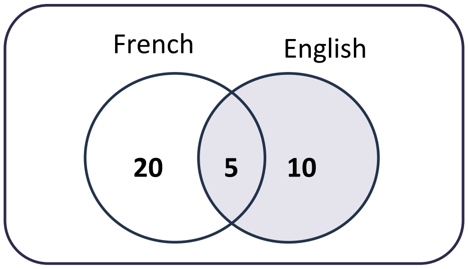
Counting using Venn forms

Exercises:

(1)

Number of people who choose to learn

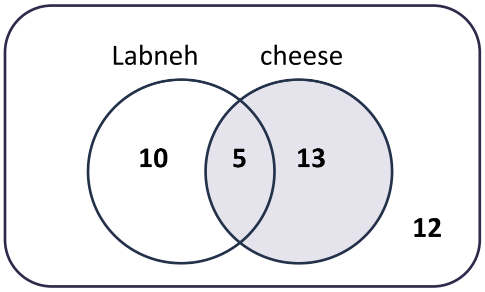
French $= 35 - 15 = 20$



(2)

Number of people who prefer both types

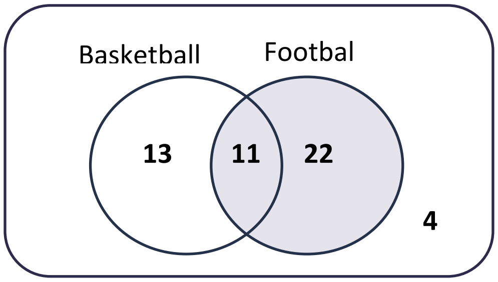
$= 12 + 18 + 15 - 40 = 5$



(3)

Number of people who don't like either game

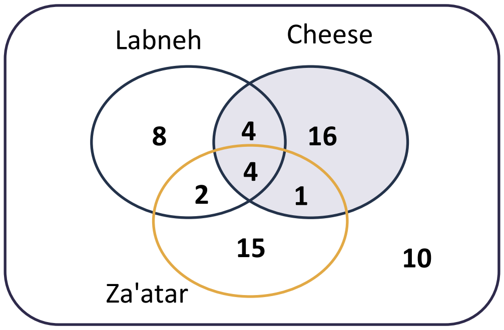
$= 12 + 18 + 15 - 40 = 5$



(4)

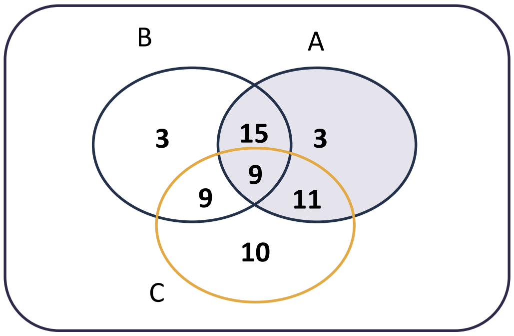
Number of people who prefer labneh =

$$60 - (16 + 1 + 15 + 10) = 18$$



(5)

Number of people who liked product (C) only = $39 - (11 + 9 + 9) = 10$



Counting a list of numbers

(6)

A simple list with 1440 numbers

(7)

-7	$8, 9, \ldots \ldots, 2019$
	$1, 2, \ldots \ldots, 2012$

A simple list with 2012 numbers

(8)

-1	$5, 9, 13, \ldots \ldots, 505$
$\div 4$	$4, 8, 12, \ldots \ldots, 504$
	$1, 2, 3, \ldots \ldots, 126$

A simple list with 126 numbers

(9)

$\div 2$	$2, 4, 6, \ldots \ldots, 2024$
	$1, 2, \ldots \ldots, 1012$

A simple list with 1012 numbers

(10)

$*\ 7$	$\frac{3}{7}, 1, \frac{11}{7} \dots \dots .., 289$
$+1$	$3, 7, 11, \dots \dots .., 2023$
$\div\ 4$	$4, 8, 12, \dots \dots .., 2024$
	$1, 2, 3, \dots \dots .., 506$

A simple list with 506 numbers

(11)

-63	$66, 69, \dots \dots .., 2022$
$\div\ 3$	$3, 6, \dots \dots .., 1959$
	$1, 2, 3, \dots \dots .., 653$

A simple list with 653 numbers

(12)

$$24^2 = 576 < 600\ , 25^2 = 625 > 600$$

So, the larger full square is less than 600 is $24^2 = 576$

$\sqrt{\ }$	$1^2, 2^2, \dots \dots .., 24^2$
	$1, 2, \dots \dots .., 24$

A simple list with 24 numbers

(13)

Common multiples of $\{5,3\}$ are the multiples of 15, And the biggest multiplier of 15 less than 1447 is 1440

$\div 15$	$15, 30, \ldots \ldots, 1440$
	$1, 2, \ldots \ldots, 96$

A simple list with 96 numbers

Solution 2:

$$\left\lfloor \frac{1447}{15} \right\rfloor = 96$$

(14)

multiples of *7*

	7,14, … … . . ,2023
÷ 7	1,2, … … . . ,289

L.C.M(5,7) =35

	35,70, … … . . ,1995
÷ 35	1,2, … … . . ,57

So, Number of multiples of *7* and not *5* is $289 - 57 = 232$

Solution 2:

$$\left\lfloor \frac{2025}{7} \right\rfloor - \left\lfloor \frac{2025}{35} \right\rfloor = 232$$

(15)

multiples of 7

$$\begin{array}{r|l} \div 7 & 7,14, \ldots \ldots ..,497 \\ \hline & 1,2, \ldots \ldots ..,71 \end{array}$$

$$L.C.M(2,7) = 14$$

$$\begin{array}{r|l} \div 14 & 14,28, \ldots \ldots ..,490 \\ \hline & 1,2, \ldots \ldots ..,35 \end{array}$$

So, Number of multiples of 7 and not even is $71 - 35 = 36$

Solution 2:

$$\left\lfloor \frac{500}{7} \right\rfloor - \left\lfloor \frac{500}{14} \right\rfloor = 36$$

The Basic Principles of Counting1

(16)

$6 \cdot 4 = 24$

(17)

$9 \cdot 10 \cdot 10 = 900$

(18)

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$

(19)

$9 \cdot 10 = 90$

The Basic Principles of Counting2

(20)

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 17280$

(21)

$7 + 10 = 17$

(22)

$3 \cdot 5 + 4 \cdot 5 + 4 \cdot 3 = 47$

(23)

$$3 \cdot 4 \cdot 9 \cdot 8 \cdot 7 = 6048$$

(24)

$$8 \cdot 20 + 13 \cdot 20 + 8 \cdot 13 = 524$$

(25)

$a)\ 9 \cdot 5 = 45$

$b)\ 3 \cdot 5 = 15$

$c)\ 2 \cdot 4 = 8$

$d)\ 4 \cdot 3 + 5 \cdot 2 = 22$